End-of-term exam

EE1C1 "Linear Circuits A"

Place: Drebbelweg Exam Hall 2
Date: 07-11-2025
Time: 9:00 – 11:00

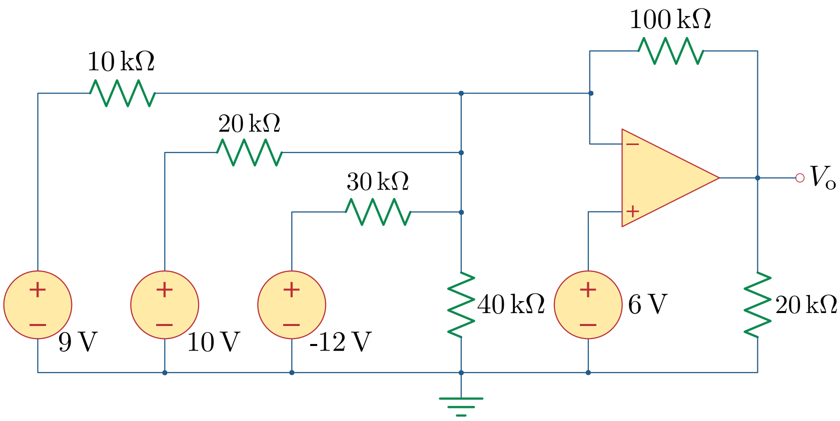
- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The exam grade is obtained by dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal/to half-a-point, after accounting for the bonus points.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the "Extra Time" (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **When requested, fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly and avoid messy solutions. Should errors occur in your solution, cross the erroneous part out and give clear indications on where the correct solution resumes.
- For this exam you are allowed to use:
    i. a simple calculator – programmable and graphic calculators are explicitly prohibited;
    ii. a handwritten, double-sided A4 sheet with formulas.
- The text of this exam is offered only in English. Inasmuch as possible, instructors will assist you with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

- Take a new double-sheet -

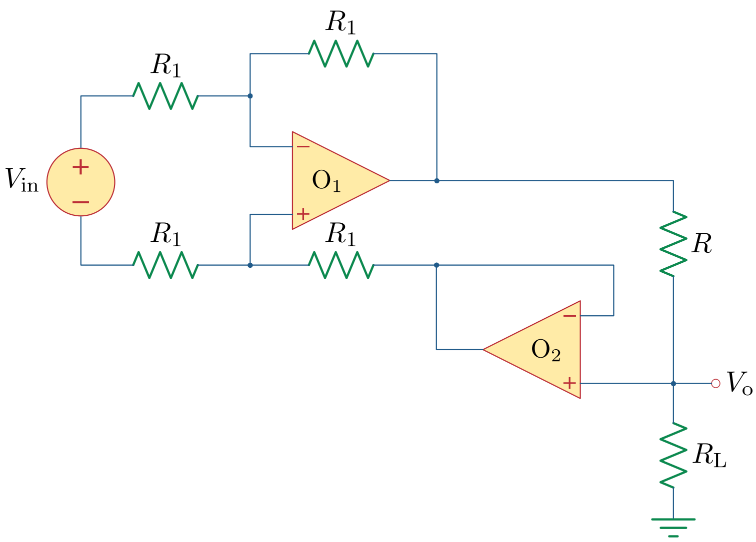
Exercise 1

Consider the circuit in the figure below, in which the op amp is taken to be ideal:



a) Calculate the output voltage V_o for the indicated input voltages. (5 points)

Now consider the circuit in the figure below, in which the op amps O_1 and O_2 are both taken to be ideal:



b) Calculate the output voltage V_o as a function of the input voltage V_{in}. (5 points)

Hints: (i) Recall that the terminal input current is always zero for ideal op amps. (ii) Examine the subcircuit comprising op amp O_2 and recognise it as a voltage follower (iii) Avoid applying KCL at the output terminal of an op amp.

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

For ideal op-amp: Both input terminals of the op-amp have the same voltage and zero input currents.

1- $V^- = V^+ = 6V$

2- $i^- = i^+ = 0A$

3- Applying KCL at the -ve input terminal of the op-amp yields:

$$(6 - 9)/10 + (6 - 10)/20 + (6 + 12)/30 + (6 - V_o)/100 + (6 - 0)/10 = 0$$

$$Vo = 31V$$

Sub-point (b)

For ideal op-amps: Both input terminals of the op-amp have the same voltage and zero input currents.

1- $V^- = V^+$

2- $i^- = i^+ = 0A$

3- Applying KVL at the input terminals of O1 the op-amp yields:

$I_{in} = V_{in} / 2R_1 = (V_x - V_z) / R_1 = - (V_x - V_y) / R_1$

$\therefore (V_x - V_z) = V_{in} / 2$ ---------------------- ①

And $(V_y - V_x) = V_{in} / 2$ ---------------------- ②

$V_y = V_o$ ---------------------- ③

Also, $(V_z - V_o) / R = V_o / R_L$ ---------------------- ④

From ①, ②, ③ and ④ : $V_{in} = V_o - V_z = -V_o R / R_L$ ------------------- ⑤

$$V_o / V_{in} = - R_L / R$$

- Take a new double-sheet -

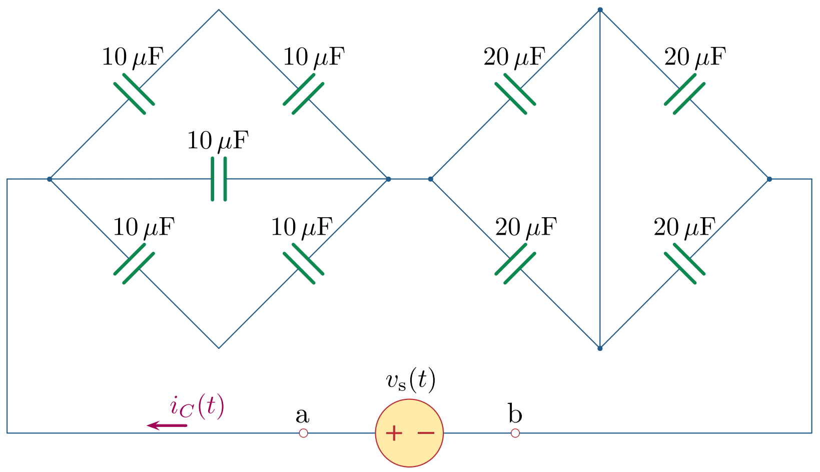
Exercise 2

The voltage at the terminals of a 100 mH inductor has the expression:

$$v_L(t) = 0.1e^{-t} + 0.2t \text{ (V)}$$

a) By knowing that $i_L(0) = 0$ A, determine the expression of the current $i_L(t)$ at $t = 2$ s. (2 points)

Consider the circuit in the figure below:



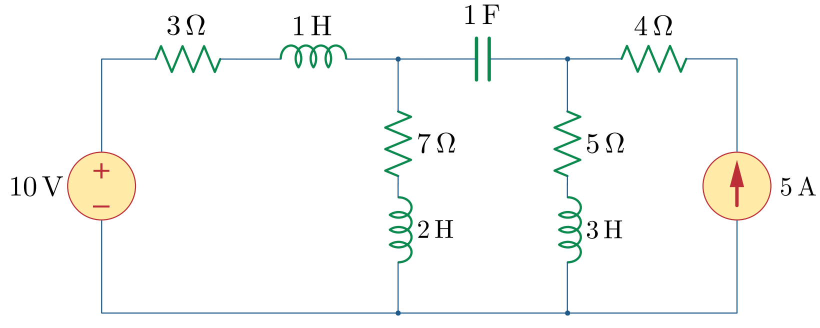
b) Calculate the equivalent capacitance C_{eq} at the terminals a–b. (2 points)

c) By knowing that the voltage source has the expression

$$v_s(t) = 3t^3 + 2t^2 + 3 \text{ (V)}$$

calculate the current $i_C(t)$ through the capacitor network at $t = 2$ s. (2 points)

Now consider the new circuit in the figure below, **operating under DC steady state conditions**:



d) Calculate the total energy w_{tot} stored in the capacitor and the three inductors. (4 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

The inductor current follows by integrating the voltage at its terminals as:

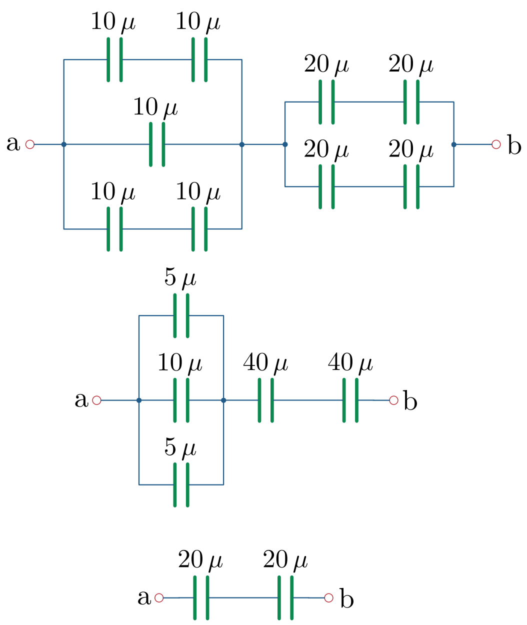
$$i_L(t) = \frac{1}{0.1}\int_{\tau=0}^{t} \left(0.1e^{-\tau} + 0.2\tau\right) \mathrm{d}\tau + i(0) = -e^{-t} + 1 + t^2$$
$$= -e^{-t} + t^2 + 1\,(\mathrm{A})$$

where the initial condition that $i_L(0) = 0$ A was also accounted for. By filling in the time $t = 2\,\mathrm{s}$, the relevant current becomes

$$i_L(2) = 4.87\,(\mathrm{A})$$

Sub-point (b)

The given circuit is successively transformed as



that, clearly, yields the equivalent capacitance

$$C_{\mathrm{eq}} = 10\,\mu\mathrm{F}$$

Sub-point (c)

The capacitor current follows from the voltage at its terminal via time-differentiation as

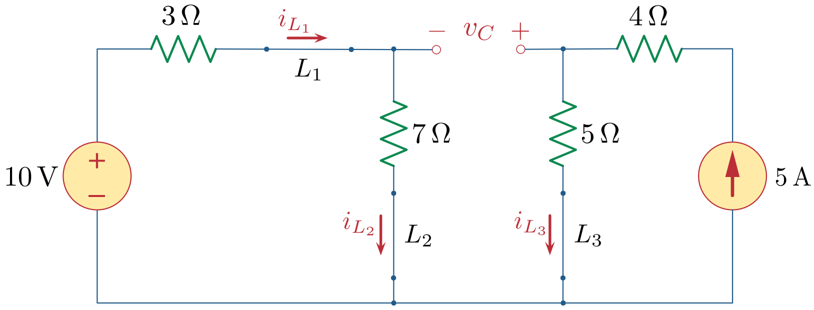
$$i_C(t) = C\frac{\mathrm{d}v_s}{\mathrm{d}t} = 10\cdot 10^{-6}\left(9t^2 + 4t\right)\,(\mathrm{A})$$

By filling in the time $t = 2\,\mathrm{s}$, the relevant current becomes

$$i_C(2) = 440\,\mu\mathrm{A}$$

Sub-point (d)

Under DC conditions, the circuit becomes:



Based on this schematic, the calculation of the needed currents/voltages is elementary:

$$i_{L_1} = i_{L_2} = \frac{10}{3+7} = 1\,(\mathrm{A})$$

$$i_{L_3} = 5\,\mathrm{A}$$

$$v_C = 5 \times i_{L_3} - 7 \times i_{L_2} = 5 \times 5 - 7 \times 1 = 18\,(\mathrm{V})$$

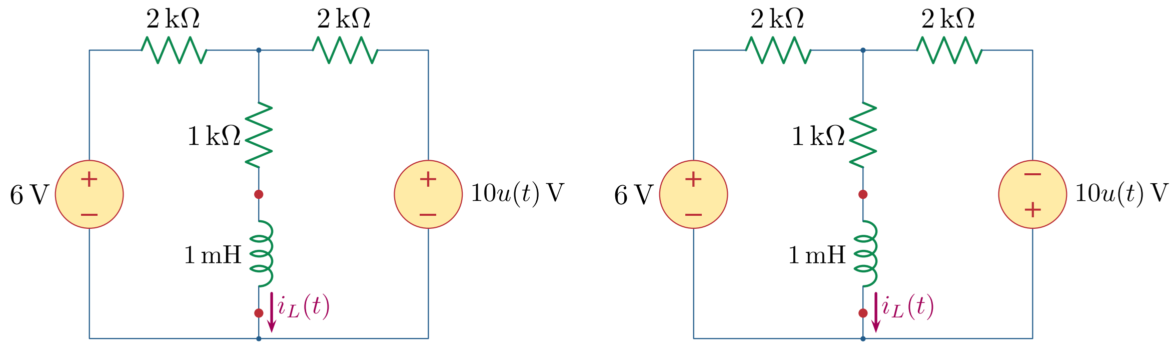
These quantities are then used in the expressions of the energy stored in the relevant circuit elements, the total energy being

$$\begin{aligned} w_{\mathrm{tot}} &= \frac{Cv_C^2}{2} + \frac{L_1 i_{L_1}^2}{2} + \frac{L_2 i_{L_2}^2}{2} + \frac{L_3 i_{L_3}^2}{2} \\ &= \frac{1 \times 18^2}{2} + \frac{1 \times 1^2}{2} + \frac{2 \times 1^2}{2} + \frac{3 \times 5^2}{2} = 201\,(\mathrm{J}) \end{aligned}$$

- Take a new double-sheet -

Exercise 3

Consider the circuit in subfigure (a) of the figure below:



a b

a) Calculate the value of the current $i_L(0^+)$. (2 points)

b) Calculate the value of the current $i_L(\infty)$. (2 points)

c) Calculate the circuit's time constant τ. (2 points)

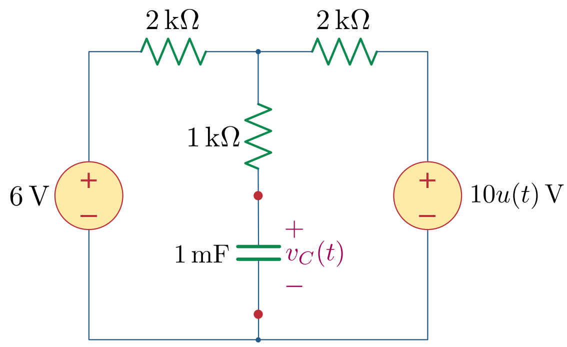
d) Determine the expression of the current $i_L(t)$ for $t > 0$. (1 point)

Assume now that the polarity of the $10u(t)$ voltage source is reversed, as in subfigure (b).

e) Select which ***one*** of the following statements applies: (i) only the time constant τ will change; (ii) only the value of the current $i_L(\infty)$ will change; (iii) both parameters will change; (iv) none of the two parameters will change. (1 point)

Hint: Please ***write down*** the statement that you deem correct, and justify briefly your choice (no points will be granted if no justification is provided).

Consider now the circuit in the figure below, in which the inductance was replaced in the original circuit via a capacitance.



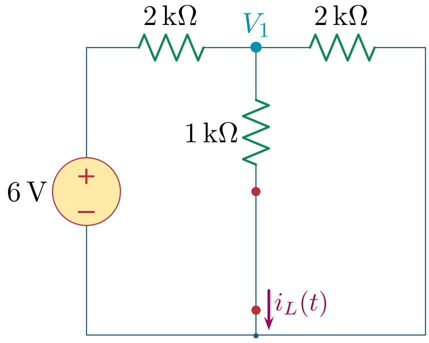
f) Calculate the new time constant τ_C. (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

The schematic for $t < 0$ is



where use was made of the fact that the source at the right has the value zero (short-circuit) for $t < 0$. It then follows consecutively that

$$R_{\|} = 1\,\text{k} \parallel 2\,\text{k} = \frac{1\,\text{k} \cdot 2\,\text{k}}{1\,\text{k} + 2\,\text{k}} = \frac{2}{3}\,(\text{k}\Omega)$$

$$V_1 = \frac{R_{\|}}{2\,\text{k} + R_{\|}} 6 = \frac{\frac{2}{3}\,\text{k}}{2\,\text{k} + \frac{2}{3}\,\text{k}} 6 = \frac{3}{2}\,(\text{V})$$

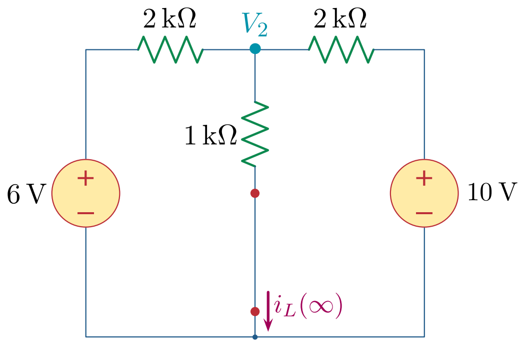
$$i_L(0^-) = \frac{V_1}{1\,\text{k}} = \frac{\frac{3}{2}}{1\,\text{k}} = \frac{3}{2}\,(\text{mA})$$

By now applying the continuity of $i_L(t)$ at $t = 0$, it immediately follows that

$$i_L(0^-) = i_L(0^+) = \frac{3}{2}\,\text{mA}$$

Sub-point (b)

The schematic for $t \to \infty$ is



By now applying nodal analysis at the node 2 it follows that

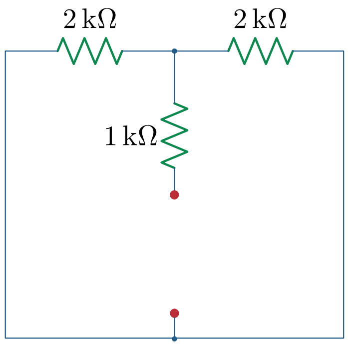
$$\frac{V_2 - 6}{2\,\text{k}} + \frac{V_2}{1\,\text{k}} + \frac{V_2 - 10}{2\,\text{k}} = 0 \implies 4V_2 = 16 \implies V_2 = 4\,\text{V}$$

and, thus

$$i_L(\infty) = \frac{V_2}{1\,\text{k}} = 4\,\text{mA}$$

Sub-point (c)

The passivised circuit at the inductance's terminals is



It is then clear that the Thévenin resistance is

$$R_{\text{Th}} = 1\,\text{k} + (2\,\text{k} \parallel 2\,\text{k}) = 1\,\text{k} + 1\,\text{k} = 2\,(\text{k}\Omega)$$

that, in turn, yields the time constant

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{1\,\text{m}}{2\,\text{k}} = 0.5\,(\mu\text{s})$$

Sub-point (d)

By now collecting all partial results in the initial-final values formula, the requested current is

$$i_L(t) = 4 + \left(\frac{3}{2} - 4\right) e^{-t/0.5\mu} = 4 - \frac{5}{2} e^{-2\cdot 10^6 t}\,(\text{mA})$$

Sub-point (e)

Since the only change in the circuit was flipping the specified source, it is evident that **the passivised circuit at the inductance's terminals will not change** and, thus, the time circuit's constant cannot change. However, flipping the source will definitely influence the final value. Consequently, the only applicable statement is: **only the value of the current $i_L(\infty)$ will change**.

$$v_L(\infty) = -1\,\text{mA}$$

Note: By employing the same reasoning as the one used at subpoint (b), it is an easy exercise to show that, in this case,

Sub-point (f)

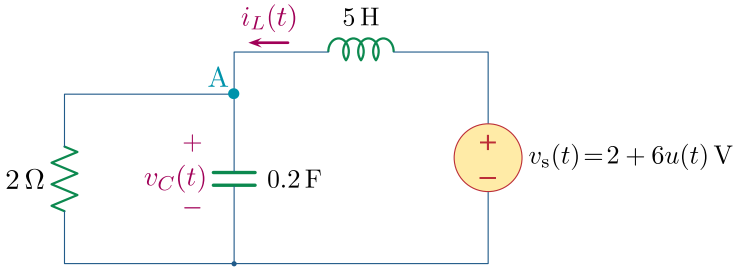
By replacing the inductance by a capacitance in the original circuit, **the passivised circuit at the reactive element's terminals will again remain the same**. As a result, calculating the requested time constant is immediate, its value being

$$\tau_C = CR_{\text{Th}} = 1\,\text{m} \cdot 2\,\text{k} = 2\,(\text{s})$$

- Take a new double-sheet -

Exercise 4

Consider the circuit in the figure below:



a) Calculate $v_C(0^+)$ and $i_L(0^+)$. (1 point)

b) Calculate $v_C(\infty)$ and $i_L(\infty)$. (1 point)

c) Apply the KCL at node A to derive an expression for the integro-differential equation for $v_C(t)$ for $t > 0$. (2 points)

d) Derive the characteristic equation and motivate if the circuit is overdamped, critically damped, or underdamped (no points will be granted if no justification is provided). (2 points)

e) Determine $v_C(t)$ for $t > 0$. (4 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

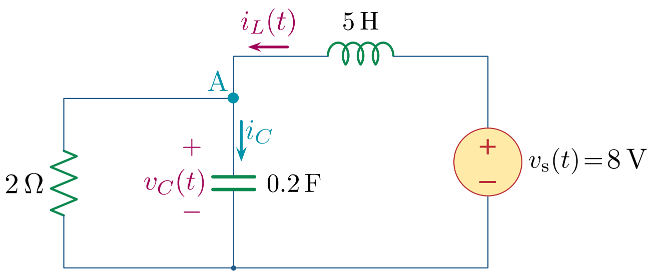
In view of continuity

$$V_C(0^-) = V_C(0^+) = \boxed{2V}$$

$$I_L(0^-) = I_L(0^+) = \frac{2V}{2\Omega} = \boxed{1A}$$

Sub-point (b)

The schematic for $t > 0$ is



AT $t \to \infty$ THE VOLTAGE SOURCE PROVIDES $V_S = 8V$ SO

$V_C(\infty) = \boxed{8V}$ AND $I_L(\infty) = \frac{8V}{2\Omega} = \boxed{4A}$

Sub-point (c)

$$i_C(t) + i_R(t) = i_L(t)$$

$$C\frac{dV_C(t)}{dt} + \frac{V_C(t)}{R} = i_L(t)$$

$$C\frac{dV_C(t)}{dt} + \frac{V_C(t)}{R} = \frac{1}{L}\int_0^t V_L(\tau)d\tau + I_L(0^+) = \frac{1}{L}\int_0^t [V_S - V_C(\tau)]d\tau + 1$$

$$C\frac{d^2V_C(t)}{dt^2} + \frac{d}{dt}\frac{V_C(t)}{R} = \frac{V_S}{L} - \frac{V_C(t)}{L} \Rightarrow \boxed{\frac{d^2V_C(t)}{dt^2} + \frac{1}{RC}\frac{dV_C(t)}{dt} + \frac{1}{LC}V_C(t) = \frac{V_S}{LC}}$$

Sub-point (d)

$$\underline{s^2 + \frac{s}{RC} + \frac{1}{LC} = 0} \qquad s^2 + \frac{s}{0.4} + 1 = 0 \qquad s^2 + 2{,}5s + 1 = 0$$

$$s_{1,2} = \frac{-2{,}5 \pm \sqrt{6{,}25 - 4}}{2}$$

$$s_1 = -2 \qquad s_2 = -0{,}5$$

OVERDAMPED

(ROOTS DISTINCT AND REAL)

Sub-point (e)

GENERAL SOLUTION FOR $V_C(t)$ IS $V_C(t) = Ae^{-2t} + Be^{-0,5t} + V_C(\infty)$

AT $t=0^+$ $V_C(0^+) = A + B + 8 = 2 \rightarrow \underline{A+B=-6}$

$$\frac{dV_C(t)}{dt} = -2Ae^{-2t} - 0{,}5Be^{-0{,}5t}$$

AT $t=0$ $\frac{dV_C(t)}{dt} = \frac{I_C}{C} = \frac{I_L - I_R}{C} = \frac{1}{C}\left(I_L(0^+) - I_R(0^+)\right) =$

$= \frac{1}{C}\left(1 - \frac{V_C(0^+)}{R}\right) = \frac{1}{C}\left(1 - 2/2\right) = 0$ SO $\underline{0 = -2A - 0{,}5B}$

$$\begin{cases} A+B=-6 \\ 0=-2A-0{,}5B \end{cases} \begin{cases} A-4A=-6 \\ B=-4A \end{cases} \begin{cases} -3A=-6 \rightarrow A=2 \\ B=-8 \end{cases}$$

$$V_C(t) = 2e^{-2t} - 8e^{-0{,}5t} + 8\,V$$

FINALLY $I_C(t) = C\frac{dV_C(t)}{dt} = 0{,}2\left(2(-2)e^{-2t} - 8(-0{,}5)e^{-0{,}5t}\right) =$

$$i_C(t) = \boxed{-0{,}8e^{-2t} + 0{,}8e^{-0{,}5t}\,A}$$