

End-of-term Exam

EE1C11 “Linear Circuits A”

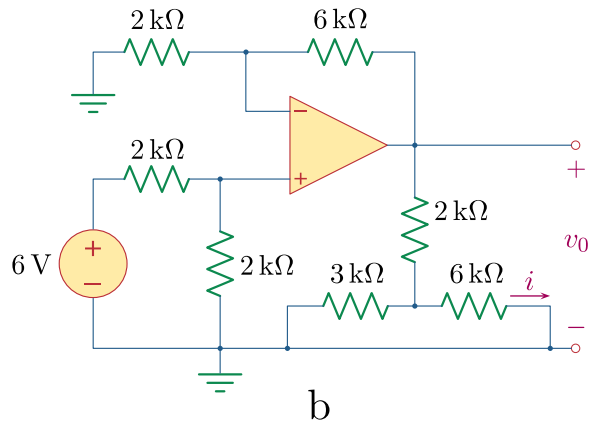
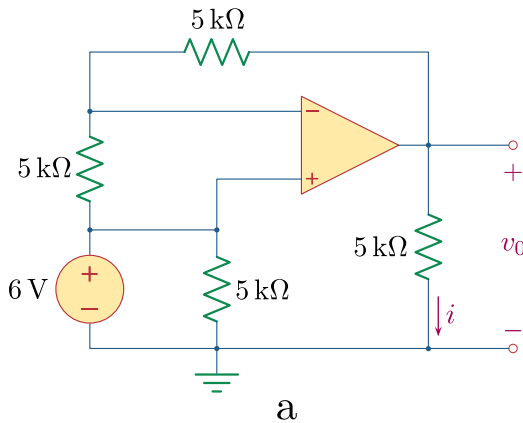
- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The grade is obtained dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **Fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly; avoid messy solutions; should errors occur in your solution, cross the erratic part out and give clear indication on where the correct solution resumes.
- For this exam you are allowed to use:
 - i. a simple calculator – programmable and graphing calculators are explicitly prohibited;
 - ii. a handwritten, double-sided A4 sheet with formulas.
- This exam is provided only in English. Instructors will provide assistance with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

- Take a new double-sheet -

Exercise 1

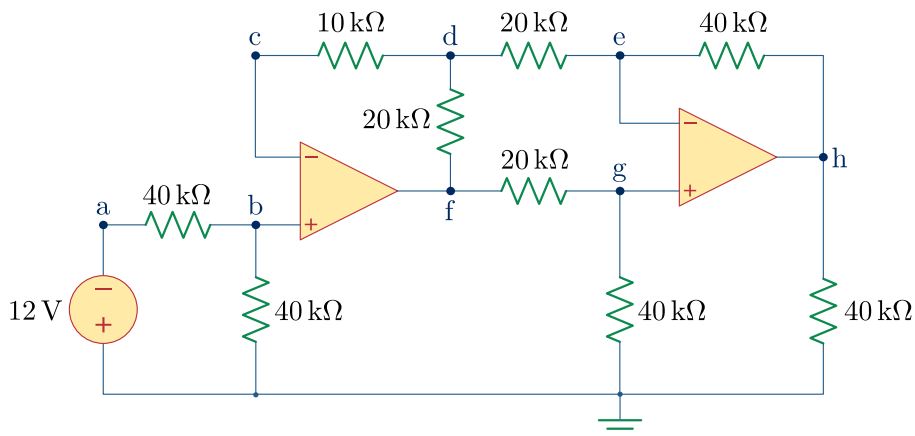
Consider the circuits in the subfigures below:



a) Calculate the output voltage v_0 and the current i in the circuit in subfigure (a), by assuming the op amp to be ideal. (2 points)

b) Calculate the output voltage v_0 and the current i in the circuit in subfigure (b), by assuming the op amp to be ideal. (3 points)

Now consider the circuit in the figure below:



c) Calculate the nodal voltages a – h in the circuit, by assuming both op amps to be ideal. (5 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

KVL:

$$-6 + V_d + V_{R_b} + V_o = 0$$

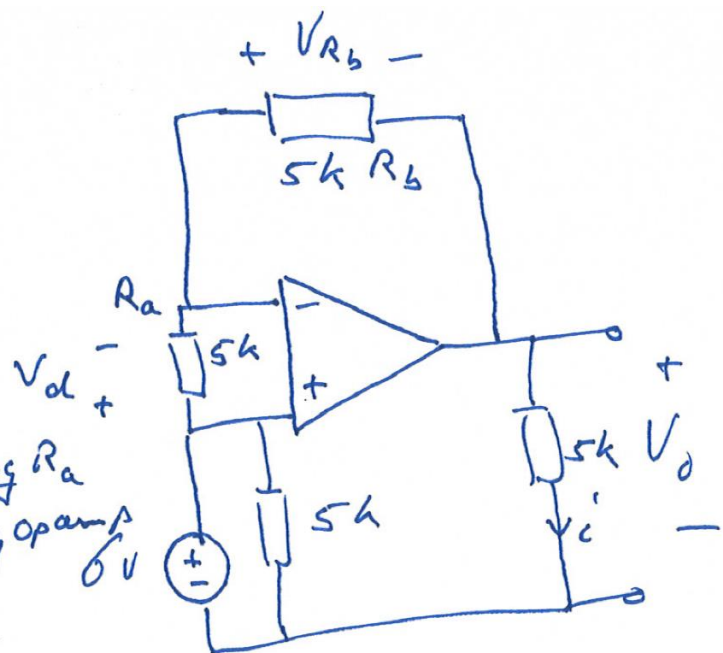
$$V_d = 0 \text{ (ideal opamp)}$$

$V_{R_b} = 0$ no current passing R_a
no current entering opamp
(ideal opamp).

therefore no
current passing R_b

$$-6 + 0 + 0 + V_o = 0 \Rightarrow V_o = 6$$

$$i = \frac{6}{5k} = 1.2 \text{ mA}$$



Sub-point (b)

non inverse amplifier: $V_o = V_+ \left(1 + \frac{6k}{2k}\right) = 4V_+$

$$V_+ = \frac{6 \cdot 2k}{2k + 2k} = 3V \Rightarrow V_o = 12V$$

$$i_{load} = \frac{12}{2k + 3k \parallel 6k} = \frac{12k}{4k} = 3 \text{ mA} \quad i = i_{load} \cdot \frac{3k}{3k + 6k} = 1 \text{ mA}$$

Sub-point (c)

$$\boxed{V_a = -12V} ; \boxed{V_b = \frac{1}{2} V_a = -6V} ; \boxed{V_c = V_b = -6V} \text{ (ideal opamp)}$$

$$\boxed{V_d = V_c = -6V} \text{ (ideal opamp, no current into opamp so no current passing } 10k\Omega \text{ between d and c)}$$

Nodal at point D:

$$\frac{V_d - V_c}{10k} + \frac{V_d - V_e}{20k} + \frac{V_d - V_f}{20k} = 0 \quad \left\{ \begin{array}{l} 0 + \frac{-6 - V_f \cdot \frac{2}{3}}{20k} + \frac{-6 - V_f}{20k} = 0 \\ \Rightarrow \boxed{V_f = -7.2V} \Rightarrow \boxed{V_g = -4.8V} \\ \boxed{V_e = V_g = -4.8V} \end{array} \right.$$

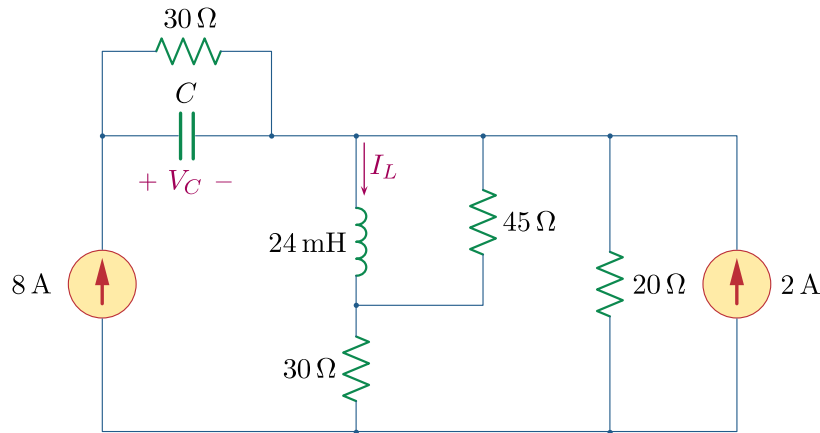
$$V_e = V_g = V_f \cdot \frac{40k}{40k + 20k} = V_f \cdot \frac{2}{3}$$

Nodal at e: $\frac{V_e - V_d}{20k} + \frac{V_e - V_a}{40k} + 0 = 0 \Rightarrow \boxed{V_A = -2.4V}$

↳ no current into opamp

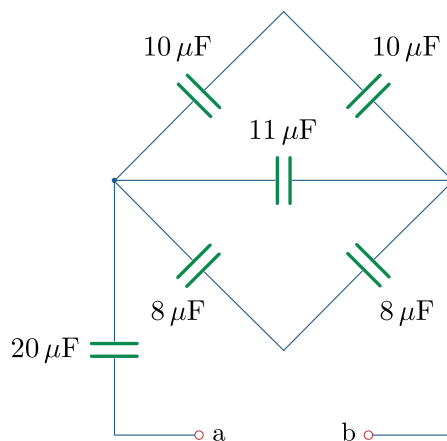
- Take a new double-sheet -

Exercise 2



Consider the circuit in the figure below under DC steady-state conditions:

- a) Calculate the current flowing through the inductance I_L and the voltage across the capacitance V_C . (3 points)
- b) Calculate the value of the capacitance C so that the energy stored in the capacitance is equal to the energy stored in the inductance. (2 points)
- c) Now consider the network of capacitors in the figure below. Calculate the equivalent capacitance C_{eq} between the terminals a–b. (2.5 points)



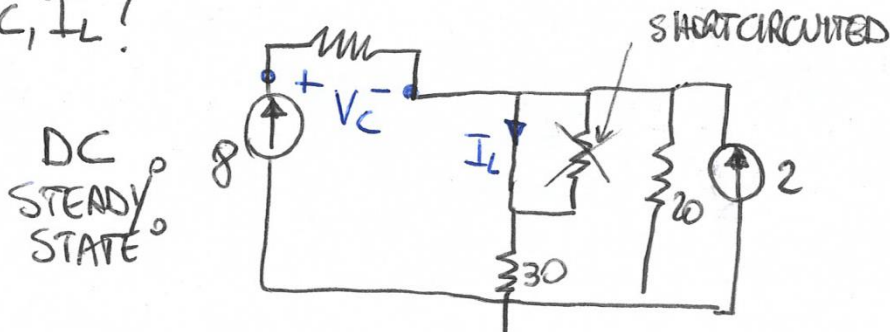
- d) Consider now a capacitance $C = 2\mu\text{F}$. Calculate the value of the current $i(t)$ at $t = 2\text{s}$ if the voltage across this capacitance is $v(t) = 4t^3 + 2t^2 + 6 \text{ (V)}$. (2.5 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

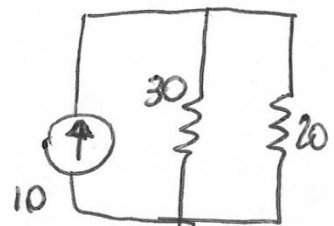
Sub-point (a)

FIND V_C, I_L ?



$$V_C = R \cdot I \text{ OHM'S LAW} \rightarrow V_C = 30 \cdot 8 = \underline{240V} \checkmark$$

$$I_L = \frac{(8+2) \cdot 20}{20+30} = \frac{10 \cdot 20}{50} = \underline{4A} \text{ CURRENT DIVIDER} \checkmark$$



Sub-point (b)

FIND C TO EQUATE ENERGY ωL

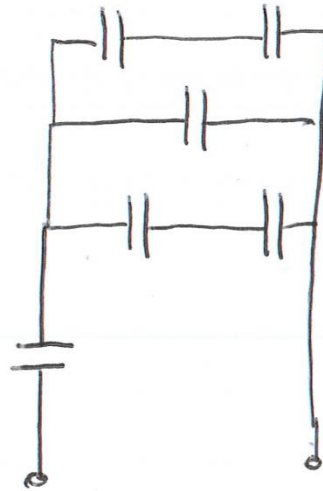
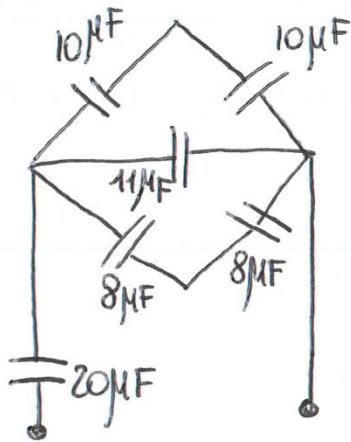
$$\frac{1}{2} C V^2 = \frac{1}{2} L I^2$$

$$C = \frac{L I^2}{V^2} = \frac{24 \cdot 10^{-3} \cdot 16}{240 \cdot 240} = \frac{24 \cdot 16 \cdot 10^{-3}}{24 \cdot 24 \cdot 10^2}$$

$$= \frac{2}{3} \cdot 10^{-5} F = 6.67 \cdot 10^{-6} F / \underline{6.67 \mu F} \checkmark$$

Sub-point (c)

EQUIVALENT C?



$$\begin{aligned} C_{eq} &= 20 \parallel (10 \parallel 10 + 11 + 8 \parallel 8) \\ &= 20 \parallel (5 + 11 + 4) \\ &= 20 \parallel 20 = \underline{10 \mu F} \checkmark \end{aligned}$$

// OPERATOR
FOR "PARALLEL"
CALCULATION THAT IS
VALID FOR CAPACITORS
IN SERIES

Sub-point (d)

GIVEN THE VOLTAGE ACROSS A CAPACITOR, FIND THE CURRENT AT
TIME $t=2$. CAPACITOR IS $2 \mu F$

$$V(t) = 4t^3 + 2t^2 + 6$$

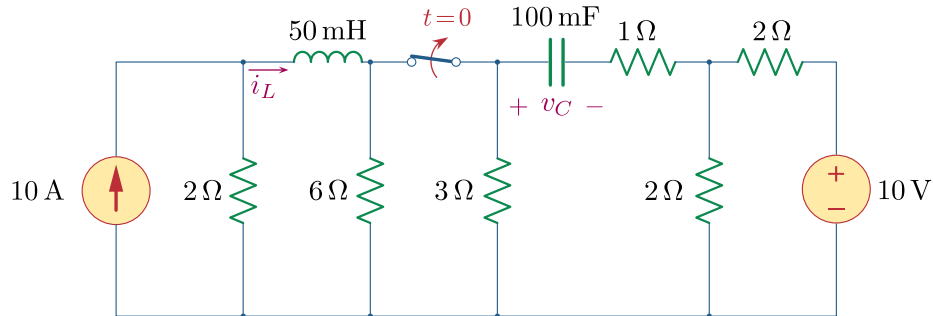
$$I(t) = C \frac{dV(t)}{dt} = 2 \cdot 10^{-6} \cdot [12t^2 + 4t]$$

$$\begin{aligned} I(2) &= 2(48 + 8) \cdot 10^{-6} = 112 \cdot 10^{-6} \text{ A} \\ &= 112 \mu A \end{aligned}$$

- Take a new double-sheet -

Exercise 3

Consider the circuit in the figure below:



The switch has been closed for a long time. At $t = 0$ the switch opens.

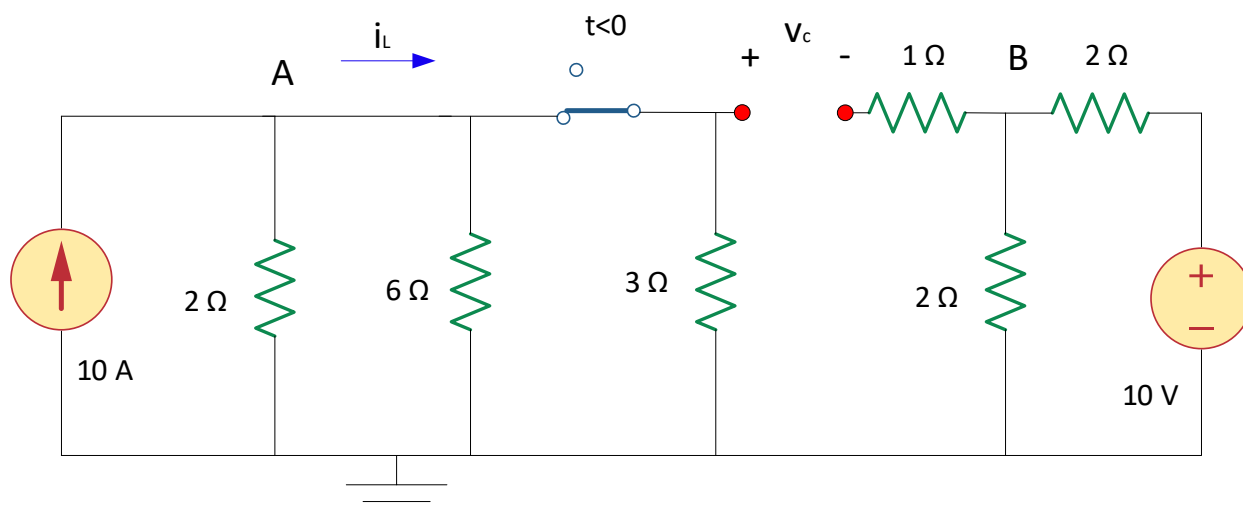
- a) Calculate the quantities $v_C(0^+)$, $i_L(0^+)$, $v_C(\infty)$ and $i_L(\infty)$. (4 points)
- b) Calculate the Thévenin resistance seen by the capacitance for $t > 0$. (3 points)
- c) Calculate $v_C(t)$ $t > 0$. (3 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

The circuit at $t = 0^-$ is as follow: capacitor is open and inductor is shorted.



The left side of the circuit we have three resistors in parallel.

$$3 \parallel 6 \parallel 2 = 1 \Omega$$

Voltage at point A is: $V_A(0^-) = IR = 10 \times 1 = 10 \text{ V}$

Voltage at point B is: $V_B(0^-) = \frac{2}{2+2} \times 10 = 5 \text{ V}$

$$v_c(0^-) = V_A - V_B = 10 - 5 = 5 \text{ V}$$

$$v_c(0^+) = v_c(0^-) = 5 \text{ V}$$

To calculate current of the inductor we should notice that it is the same as the current flowing thorough the parallel of 3 and 6 Ohm resistors:

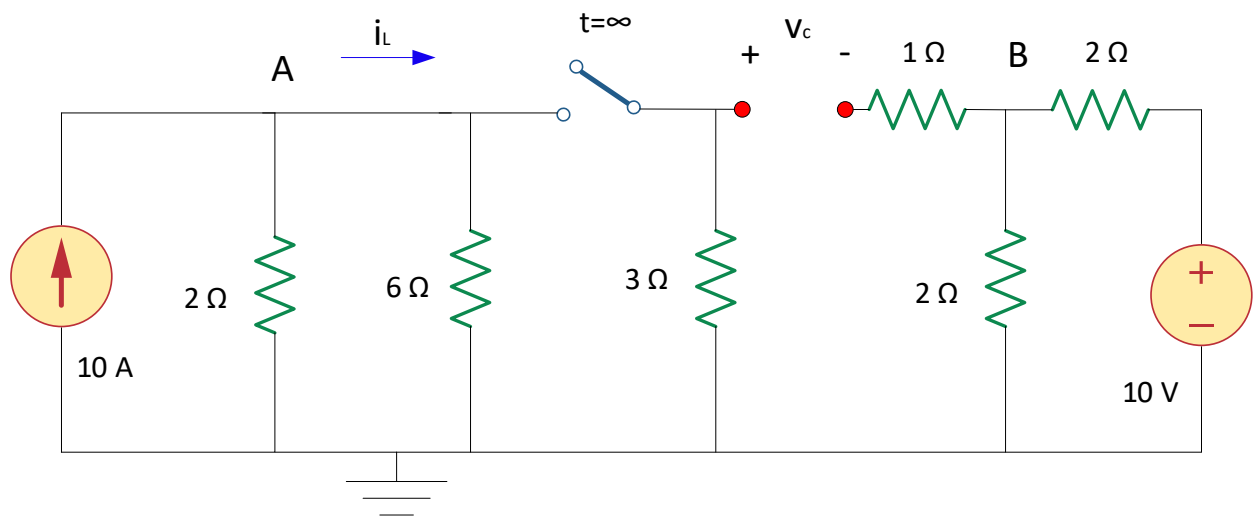
$$3 \parallel 6 = 2 \Omega$$

current through the inductor is:

$$i_L(0^-) = \frac{2}{2+2} \times 10 = 5 \text{ A}$$

$$i_L(0^+) = i_L(0^-) = 5 \text{ A}$$

The circuit at $t = \infty$ is as follows:



Using current division we can calculate the current of the inductor:

$$i_L(\infty) = \frac{2}{2+6} \times 10 = 2.5 \text{ A}$$

Voltage at point B is: $V_B(\infty) = \frac{2}{2+2} \times 10 = 5 \text{ V}$

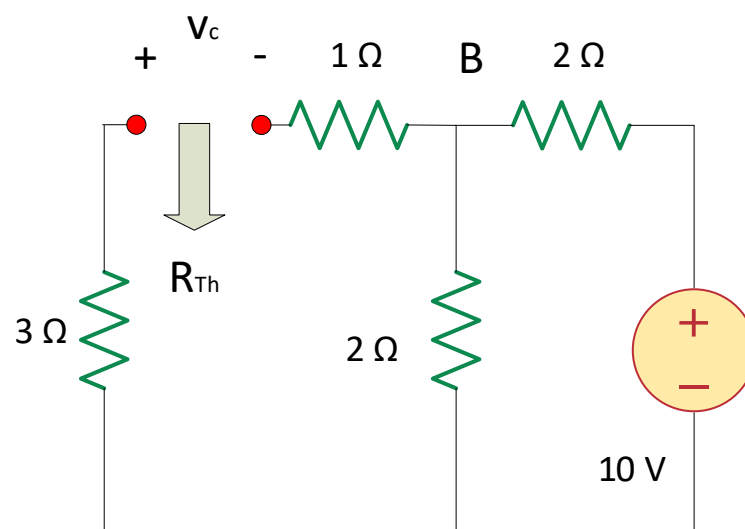
$$v_c^-(\infty) = V_B(\infty) = 5 \text{ V}$$

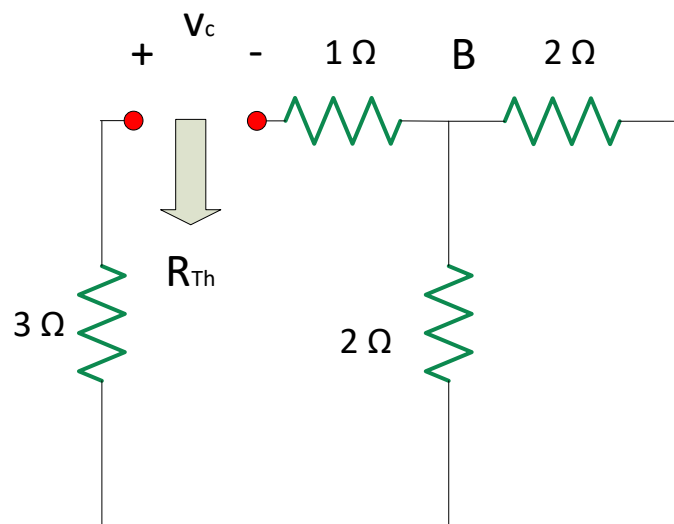
$$v_c^+(\infty) = 0 \text{ V}$$

$$v_c(\infty) = v_c^+(\infty) - v_c^-(\infty) = -5 \text{ V}$$

Sub-point (b)

To find R_{Th} we eliminate the voltage source:





$$R_{Th} = 2 \parallel 2 + 1 + 3 = 5\ \Omega$$

Sub-point (c)

$$v_c(t) = (v(0) - v(\infty))e^{-\frac{t}{\tau}} + v(\infty)$$

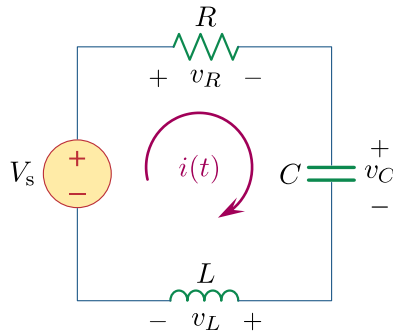
$$\tau = RC = 5 * 0.1 = 0.5$$

$$v_c(t) = (5 - (-5))e^{-\frac{t}{0.5}} - 5$$

$$v_c(t) = 10e^{-2t} - 5$$

- Take a new double-sheet -

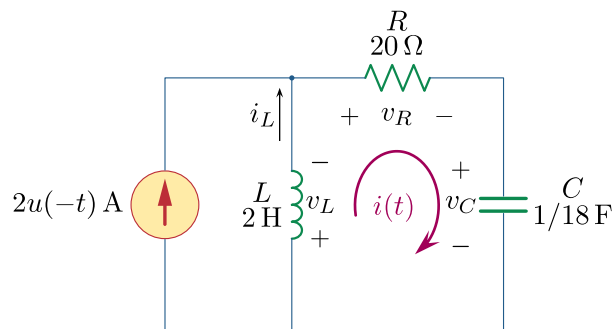
Exercise 4



Consider the circuit in the figure below:

a) Give a general *first-order* expression for $i(t)$ for $t > 0$, by applying KVL to this circuit. (2 points)

(Hint: Please derive an integro-differential equation, and not a second-order differential equation).



Now consider the circuit in the figure below:

b) Calculate $v_C(0+)$, $v_R(0+)$, $i_L(0+)$ and $v_L(0+)$. (2 points)

c) Calculate $i(t)$ for $t > 0$. (5 points)

Finally, consider the current $i(t) = (5e^{-3t} + 12e^{-12t}) u(t)$ (A).

d) Calculate for which value of t this current (practically) reaches its steady state $i(\infty)$. (1 point)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

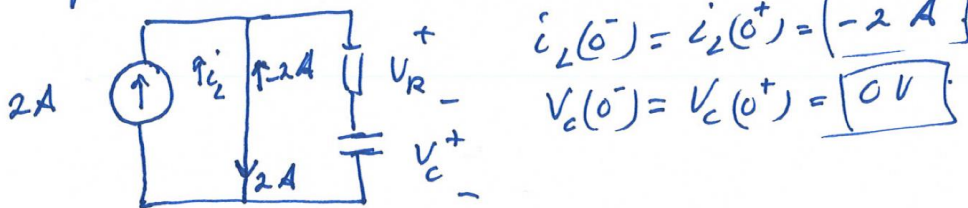
$$V_R + V_C + V_L - V_S = 0 \quad (KVL)$$

$$i(t)R + \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau + L \frac{di(t)}{dt} = V_S \quad \text{or} \quad i(t)R + \frac{1}{C} \int_0^t i(\tau) d\tau + V_C(0) + L \frac{di}{dt} = V_S$$

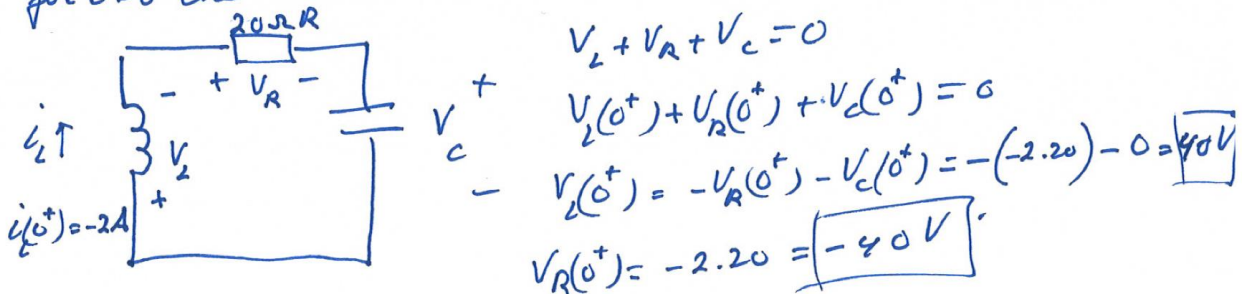
Both Uh

Sub-point (b)

for $t < 0$ the circuit becomes:



for $t > 0$ the circuit becomes:



Sub-point (c)

$$\text{for } t > 0: iR + \frac{1}{C} \int_{-\infty}^t i(z) dz + L \frac{di}{dt} = 0 \Rightarrow L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0.$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0 \Rightarrow (s^2 + 2\alpha s + \omega_0^2) = 0$$

$$\alpha = \frac{1}{2\tau} = \frac{1}{2} \frac{L}{R} = 5 \left(\frac{1}{s} \right) \quad \omega_0^2 = \frac{1}{LC} = \frac{1}{2 \cdot \frac{1}{10}} = 5$$

$$s_1, s_2: -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm \sqrt{25 - 5} = -\alpha \pm 4 = -5 \pm 4 = -1 \text{ or } -9$$

$$i(t) = A_1 e^{-t} + A_2 e^{-9t} \quad i(0) = A_1 + A_2 = -2$$

$$\frac{di}{dt} = -\frac{iR}{L} = \frac{1}{LC} \int_{-\infty}^t i(z) dz \Rightarrow \frac{di}{dt} \Big|_{t=0^+} = -\frac{i(0^+)R}{L} = V_C(0^+) = \frac{40}{2} = 20 \text{ V/s}$$

$$\Rightarrow 20 = -A_1 - 9A_2 \quad \text{Solving for } A_1 \text{ and } A_2 \Rightarrow A_1 = \frac{1}{4} \text{ and } A_2 = -\frac{9}{4}$$

$$i(t) = \frac{1}{4} e^{-t} - \frac{9}{4} e^{-9t} \quad t > 0.$$