

Mid-term Exam

EE1C11 “Linear Circuits A”

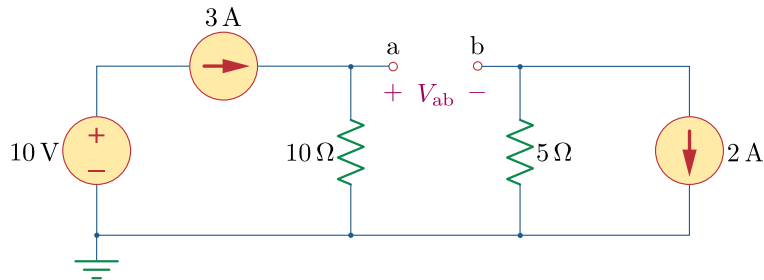
- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **Fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly; avoid messy solutions; should errors occur in your solution, cross the erratic part out and give clear indication on where the correct solution resumes.
- For this exam you are allowed to use:
 - i. a simple calculator – programmable and graphing calculators are explicitly prohibited;
 - ii. a handwritten, double-sided A4 sheet with formulas.
- This exam is provided only in English. Instructors will provide assistance with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

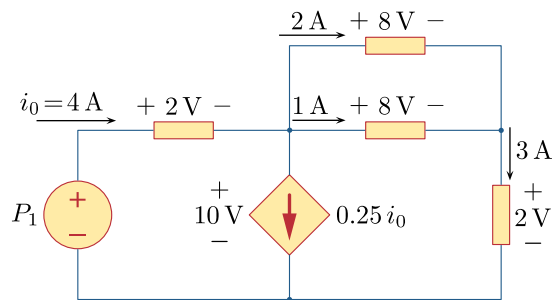
- Take a new double-sheet -

Exercise 1

a) Calculate the voltage V_{ab} between the terminals a–b in the circuit in the figure below. (4 points)



b) By making use of Tellegen's theorem, calculate the power absorbed or supplied (please specify) by the independent voltage source P_1 in the circuit in the figure below. (6 points)



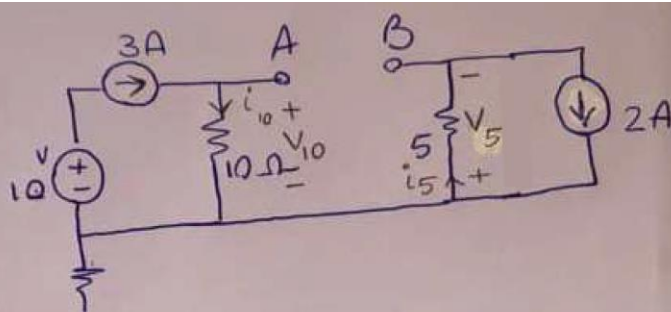
Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

$$\begin{aligned}P_2 &= +8 \text{ W} \\P_3 &= 0.25 I_o \times 10 = 10 \text{ W} \\P_4 &= 16 \text{ W} \\P_5 &= 8 \text{ W} \\P_6 &= 6 \text{ W} \\ \sum P_i &= 0 \rightarrow P_1 = -(8+10+16+8+6) \\ &= -48 \text{ W}\end{aligned}$$

Sub-point (b)



In the left loop:

the current through 10Ω resistor is $i_{10} = 3\text{A}$, which is equal to the current of the current source

Ohm law for 10Ω resistor:

$$V_{10} = V_A - 0 = R_{10} i_{10} = 10 \times 3 = 30 \text{ V} \Rightarrow V_A = 30 \text{ V}$$

In the right loop:

current through the 5Ω resistor is 2A

Ohm law for 5Ω resistor:

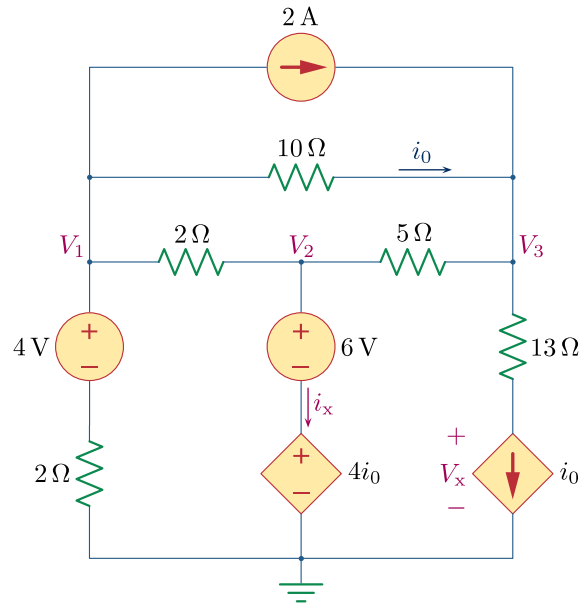
$$V_5 = 0 - V_B = R_5 i_5 = 5 \times 2 = 10 \rightarrow V_B = -10 \text{ V}$$

$$\Rightarrow V_{AB} = V_A - V_B = 30 - (-10) = 40 \text{ V}$$

- Take a new double-sheet -

Exercise 2

Consider the circuit in the figure below:



- a) Calculate via **nodal analysis** the nodal voltages V_1 , V_2 and V_3 . (6 points)
- b) Calculate the current i_x . (2 points)
- c) Calculate the voltage V_x , with the indicated polarity. (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

$$\text{KCL (1): } \frac{V_1 - 4}{2} + \frac{V_1 - V_2}{2} + 2 + i_0 = 0 \rightarrow V_1 - 4 + V_1 - V_2 + 4 + 2i_0 = 0$$

$$2V_1 - V_2 + 2i_0 = 0 \quad (1)$$

$$V_2 = 6 + 4i_0 \quad (2)$$

$$\text{KCL (3): } i_0 + \frac{V_3 - V_2}{5} - i_0 - 2 = 0 \rightarrow V_3 - V_2 = 10$$

$$V_3 - V_2 = 10 \quad (3)$$

$$\text{Ohm law: } \frac{V_1 - V_3}{10} = i_0 \rightarrow V_1 - V_3 = 10i_0$$

$$V_1 - V_3 = 10i_0 \quad (4)$$

Adding (3) and (4):

$$V_1 - V_2 = 10 + 10i_0 \quad (5)$$

Replacing V_2 from (2) in (5):

$$V_1 - 6 - 4i_0 = 10 + 10i_0 \rightarrow V_1 = 16 + 14i_0 \quad (6)$$

Replacing (6) and (2) in (1):

$$2 \times (16 + 14i_0) - (6 + 4i_0) + 2i_0 = 0 \rightarrow 26i_0 = -26 \rightarrow i_0 = -1A \quad (7)$$

$$V_1 = 2V$$

$$V_2 = 2V$$

$$V_3 = 12V$$

Sub-point (b): We write KCL at node 2:

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{5} + i_x = 0 \rightarrow 0 + \frac{2 - 12}{5} + i_x = 0 \rightarrow i_x = 2A$$

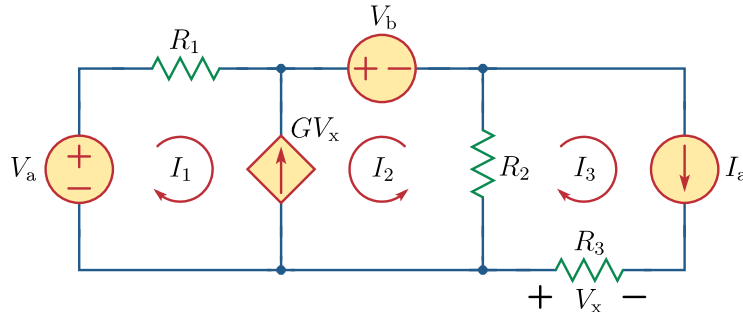
Sub-point (c): we write ohm law:

$$\frac{V_3 - V_x}{13} = i_0 \rightarrow \frac{12 - V_x}{13} = -1 \rightarrow V_x = 25V$$

- Take a new double-sheet -

Exercise 3

Consider the circuit in the figure below:



a) Using **mesh analysis**, write down all the necessary equations.

Hint: Do not fill in any numeric value! (4 points)

b) Setup a system of equations that can be solved for I_1 , I_2 and I_3 **for the given directions of the mesh currents in the figure!**

Hint: Make sure these equations only contain unknown mesh currents and constants! (3 points)

c) Now calculate I_1 , I_2 and I_3 for the following component values:

$$V_a = 5 \text{ V}$$

$$V_b = 6 \text{ V}$$

$$I_a = 2 \text{ A}$$

$$G = 2 \text{ S}$$

$$R_1 = 1 \text{ } \Omega$$

$$R_2 = 2 \text{ } \Omega$$

$$R_3 = 3 \text{ } \Omega. \text{ (3 points)}$$

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

- a) Using mesh analysis, write down all the necessary equations (do not fill in any numeric values!)

$$\begin{aligned} \text{KVL for } I_1 \text{ and } I_2: \quad I_1: R_1 I_1 + V_y - V_a &= 0 \\ I_2: +V_y + R_2 (I_2 + I_3) - V_b &= 0 \\ I_3: I_3 &= I_a \end{aligned}$$

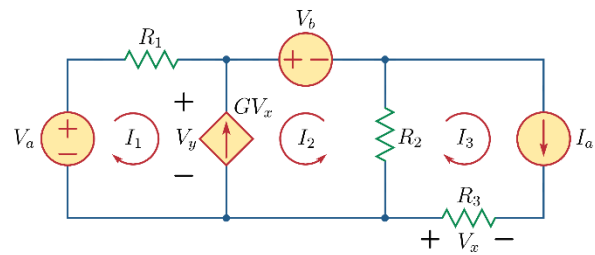
$$\text{Controlling voltage } V_x: V_x = -R_3 I_3$$

$$\text{Controlled current source: } I_1 + I_2 = -GV_x = GR_3 I_3$$

- b) Setup a system of equations that can be solved for I_1 to I_3 (make sure these equations only contain unknown mesh currents and constants!)

Eliminating V_y from eq. 1 and 2 and substituting eq. 3 gives:

$$\begin{aligned} R_1 I_1 - R_2 I_2 &= V_a - V_b + R_2 I_a \\ I_1 + I_2 &= GR_3 I_a \end{aligned} \quad \text{or in matrix form: } \begin{bmatrix} R_1 & -R_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_a - V_b + R_2 I_a \\ GR_3 I_a \end{bmatrix}$$

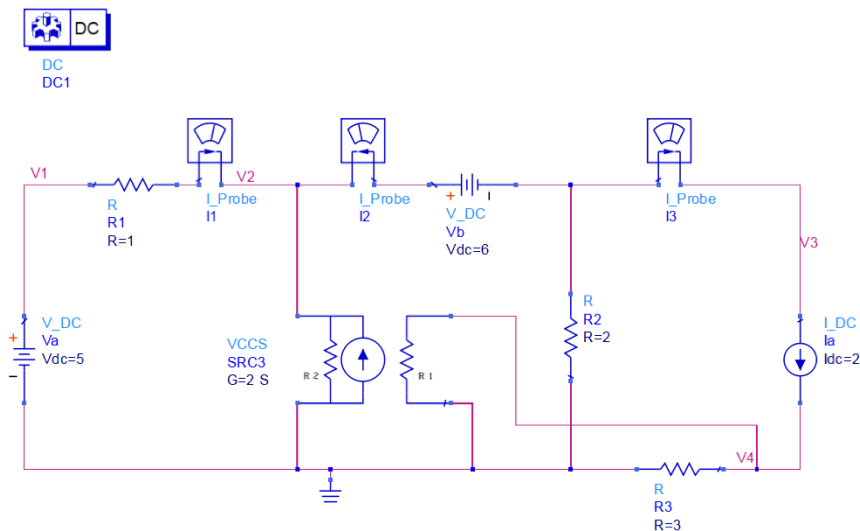


- c) Now fill in the following component values and calculate I_1 to I_3 :

$$\text{Filling in values: } \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$$

$$\text{Which gives: } I_1 = 9 \text{ A}, I_2 = 3 \text{ A} \text{ and } I_3 = 2 \text{ A}$$

$$\begin{aligned} V_a &= 5 \text{ V} \\ V_b &= 6 \text{ V} \\ I_a &= 2 \text{ A} \\ G &= 2 \text{ S} \\ R_1 &= 1 \Omega \\ R_2 &= 2 \Omega \\ R_3 &= 3 \Omega \end{aligned}$$



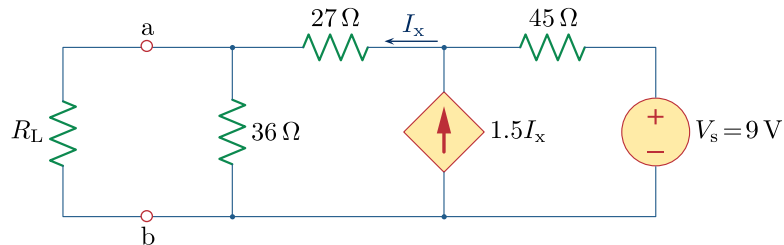
I_1	I_2	I_3
9.000 A	3.000 A	2.000 A

V_1	V_2	V_3	V_4
5.000 V	-4.000 V	-10.00 V	6.000 V

- Take a new double-sheet -

Exercise 4

Consider the circuit in the figure below:



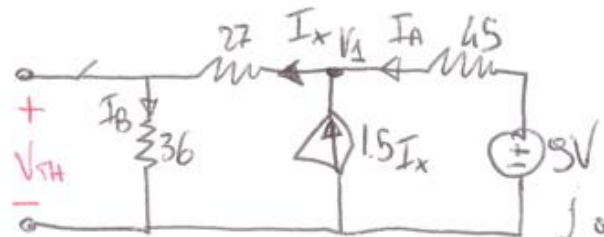
By making use of the Thévenin theorem:

- Calculate the value of the voltage V_{Th} of the circuit **at the right** of the terminals a–b. (4 points)
- Calculate the value of the resistance R_{Th} of the circuit **at the right** of the terminals a–b. (4 points)
- Find the value of the resistance R_L to achieve maximum power transfer and calculate the value of that maximum power P_{max} . (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)



NODEAL ANALYSIS

$$\begin{cases} I_A + 1.5 I_x - I_x = 0 \\ I_x = I_B \end{cases}$$

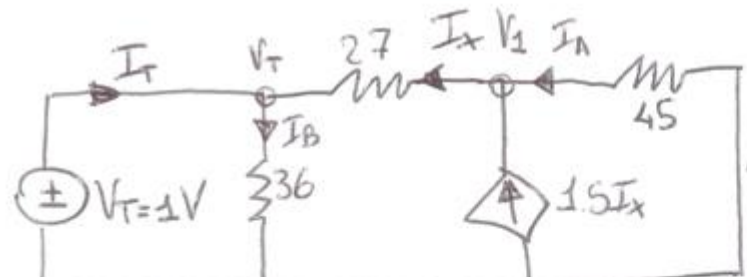
$$\begin{cases} \frac{8 - V_1}{45} + 0.5 \frac{V_1 - V_{TH}}{27} = 0 \\ \frac{V_1 - V_{TH}}{27} - \frac{V_{TH}}{36} = 0 \end{cases}$$

$$\begin{cases} 27 - 3V_1 + 2.5V_1 - 2.5V_{TH} = 0 \\ 4V_1 - 4V_{TH} - 3V_{TH} = 0 \end{cases}$$

$$\begin{cases} 27 - 0.5V_1 - 2.5V_{TH} = 0 \\ V_1 = \frac{7}{4}V_{TH} \end{cases} \begin{cases} 27 - \frac{7}{8}V_{TH} - 2.5V_{TH} = 0 \\ V_{TH} = \frac{27 \cdot 8}{27} \end{cases} \downarrow$$

$V_{TH} = 8V$

Sub-point (b)



NODEAL ANALYSIS

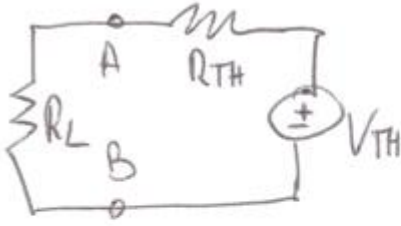
$$\begin{cases} 1.5 I_x - 0.5 I_x + I_A = 0 \\ I_x + I_T - I_B = 0 \end{cases}$$

$$\begin{cases} 0.5 I_x - \frac{V_1}{45} = 0 \\ \frac{V_1 - V_T}{27} + I_T - \frac{V_T}{36} = 0 \end{cases} \begin{cases} 0.5 \frac{V_1 - V_T}{27} - \frac{V_1}{45} = 0 \\ \frac{V_1 - V_T}{27} - \frac{V_T}{36} + I_T = 0 \end{cases} \begin{cases} 2.5V_1 - 2.5V_T - 3V_1 = 0 \\ 4V_1 - 4V_T - 3V_T + 108I_T = 0 \end{cases}$$

$$\begin{cases} V_1 = \frac{2.5V_T}{-0.5} = -5 \\ 4V_1 - 7V_T = -108I_T \end{cases} \begin{cases} -27 = -108I_T \\ I_T = \frac{27}{108} = \frac{1}{4} \end{cases}$$

$V_T = R_{TH} I_T$ $R_{TH} = V_T / I_T = 40\Omega$

Sub-point (c)



$R_L = R_{TH}$ FOR MAX POWER TRANSFER

$$P_{RL} = V \cdot I = \frac{V_{TH} \cdot R_L}{(R_L + R_{TH})} \cdot \frac{V_{TH}}{(R_L + R_{TH})}$$

$$= \frac{64 \cdot 4}{8 \cdot 8} = \boxed{4W}$$