Mid-term Exam

EE1C1 “Linear Circuits A”

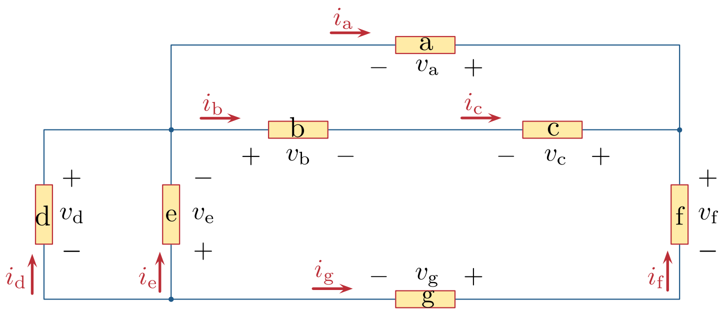
- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The exam grade is obtained by dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **When requested, fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly and avoid messy solutions. Should errors occur in your solution, cross the erroneous part out and give clear indications on where the correct solution resumes.
- For this exam you are allowed to use:
    i. a simple calculator – programmable and graphic calculators are explicitly prohibited;
    ii. a handwritten, double-sided A4 sheet with formulas.
- The text of this exam is offered only in English. Inasmuch as possible, instructors will assist you with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

- Take a new double-sheet -

Exercise 1

Consider the circuit composed of several circuit elements connected as depicted in the schematic below. Given the values in the table:



Element	Voltage (V)	Current (A)
a	1	1
b	0.5	-2
c	1.5	–
d	2	-0.5
e	–	-0.5
f	5	1
g	-2	1

a) Determine the missing values in the table. (2 points)

Hint: At subpoints (a) and (b) you may either fill in the values in a table (that you must prepare on the exam sheet) or write them as text, by clearly indicating the quantity to which each numerical value corresponds to.

b) Determine the power absorbed/supplied by each element. (5 points)

c) Determine which elements (a, b, c, d, e, f, or g) supply power. (1 point)

d) Determine the total power ***absorbed*** by the circuit. (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

Since the elements d and e are in parallel with opposite polarity: $V_{\mathrm{e}} = \text{-}V_{\mathrm{d}} = \text{-}2\mathrm{V}$. Since the current I_{b} and I_{c} are in the same branch and direction $I_{\mathrm{b}} = I_{\mathrm{c}} = \text{-}2\mathrm{A}$.

Sub-point (b)

Passive sign convention yields:

$P_{\mathrm{a}} = \text{-}I_{\mathrm{a}}V_{\mathrm{a}}$ $\quad$ $\mathrm{P}_{\mathrm{e}} = I_{\mathrm{e}}V_{\mathrm{e}}$

$P_{\mathrm{b}} = I_{\mathrm{b}}V_{\mathrm{b}}$ $\quad$ $\mathrm{P}_{\mathrm{f}} = \text{-}\ I_{\mathrm{f}}V_{\mathrm{f}}$

$P_{\mathrm{c}} = \text{-}\ I_{\mathrm{c}}V_{\mathrm{c}}$ $\quad$ $\mathrm{P}_{\mathrm{g}} = \text{-}I_{\mathrm{g}}V_{\mathrm{g}}$

$P_{\mathrm{d}} = \text{-}\ I_{\mathrm{d}}V_{\mathrm{d}}$

The resulting values are in the table:

Element	Voltage (V)	Current (A)	Power (W)	Type
a	1	1	**-1**	Supply
b	0.5	-2	**-1**	Supply
c	1.5	-2	**3**	Absorb
d	2	-0.5	**1**	Absorb
e	-2	-0.5	**1**	Absorb
f	5	1	**-5**	Supply
g	-2	1	**2**	Absorb

Sub-point (c)

The elements supplying power are a, b, f.

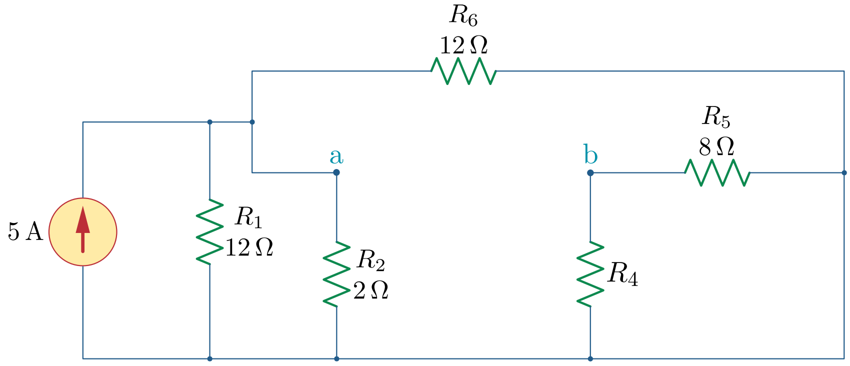
Sub-point (d)

$P_{\mathrm{absorbed}} = 3 + 1 + 1 + 2 = 7\mathrm{W}$

- Take a new double-sheet -

Exercise 2

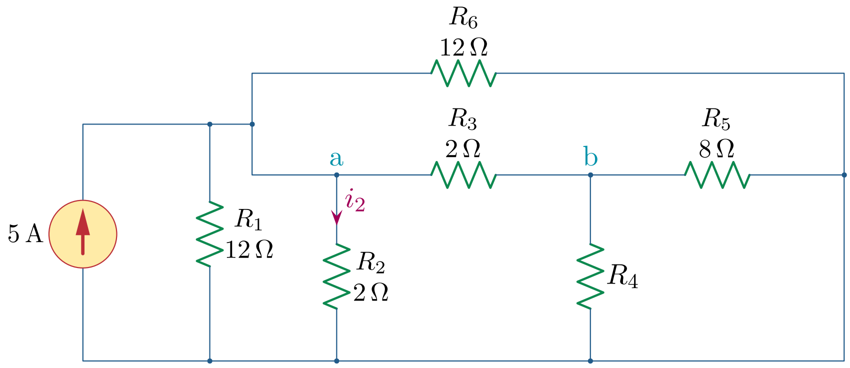
Consider the circuit in the figure below:



a) Determine the power delivered by the 5A current source. (3 points)

Hint: Redraw the circuit, by carefully observing the circuit elements that share the same voltage, or the same current.

A 2Ω resistance R_3 is now connected between the nodes a-b, as in the figure below:



b) Determine the value of the resistance R_4 by knowing that the current flowing through $R_2 = 2\Omega$ is now $i_2 = 3\text{A}$. (4 points)

Hint: Redraw the circuit in a simplified form, by carefully observing the circuit elements that share the same voltage, or the same current. Examine then the thus simplified circuit.

c) Determine the power delivered by the 5A current source in this new configuration. (3 points)

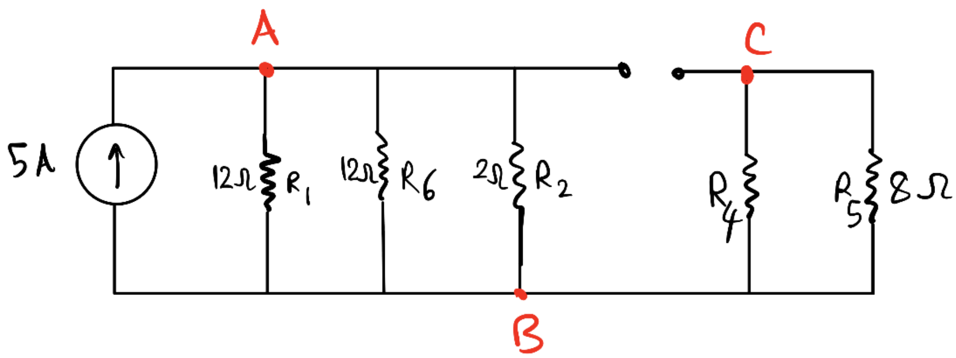
Hint: Use $R_4 = 8/7\Omega$ if you did not succeed in obtaining a value for R_4 at subpoint (b) – this is not the solution of the exercise!

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

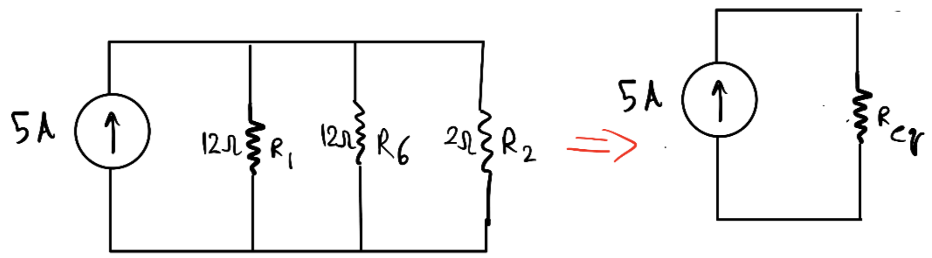
Solution

Sub-point (a)

By observing the circuit elements that share the same voltage, the given circuit can be redrawn as



Clearly, R_4 and R_5 are not part of the circuit and we are left with 3 resistances in parallel. With this in mind, the reminder of the solution is straightforward.



Calculating the equivalent resistor R_{eq}:

$$R_{eq} = 12 \| 12 \| 2 = 6 \| 2 = \frac{2\times 6}{8} = \boxed{1.5\,\Omega}$$

using tellegen (or an equivalent step)

$$P_{source} + P_{Req} = 0 \Rightarrow P_{Source} = -P_{Req}$$

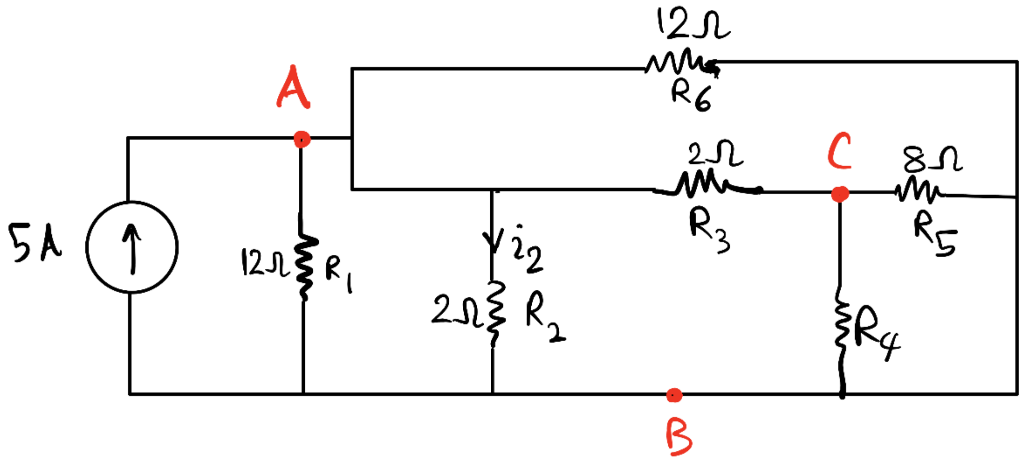
$$P_{Req} = +R_{eq} I_s^2 = 1.5 \times 5^2 = \boxed{37.5\,W}$$

then $$P_{Source} = \boxed{-37.5\,W}$$

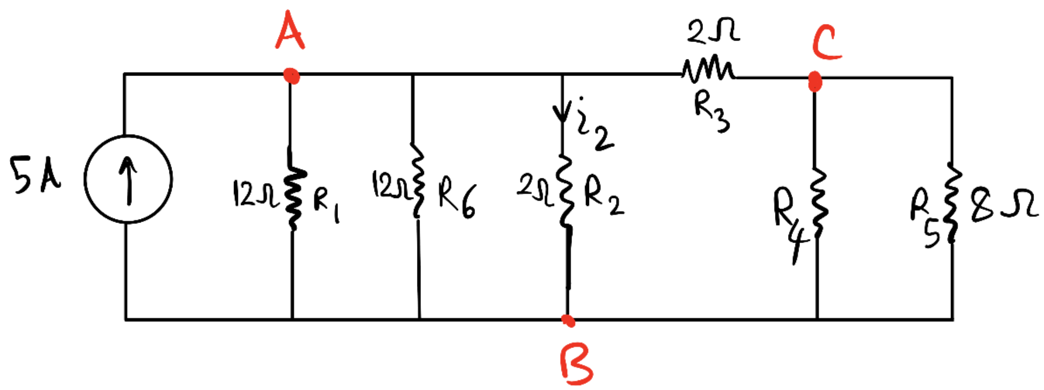
Note that this result assumes the passive sign convention for the source. If an active sign were used (as it may be suggested by "delivered") the sign would be plus.

Sub-point (b)

This point requires an attentive analysis of the circuit. The important nodes to focus upon are A, B and C in the figure below:



Examining those nodes shows that the new circuit has important similarities with the previous: we have the same 3 resistances in parallel and a new group consisting of $R_3 + (R_4 \parallel R_5)$ which is also in parallel with the previous group (see the new schematic below).



With these observations, the solution is now starts by firstly using current division:

if $i_2 = 3A$ using current division:

$$\frac{3}{5} = \frac{R_{eq}}{\cancel{R_2} + R_{eq}}$$

2Ω

where $R_{eq} = (R_1 \parallel R_6) \parallel [R_3 + R_4 \parallel R_5]$

$$\Rightarrow \boxed{R_{eq} = 3\,\Omega}$$

and then proceeding with

$$3\,\Omega = (12 \parallel 12) \parallel [8 \parallel R_4 + 2]$$

$$\rightarrow \quad 3 = 6 \parallel [\; R_{eq2} \;]$$

where $R_{eq2} = 8 \parallel R_4 + 2$

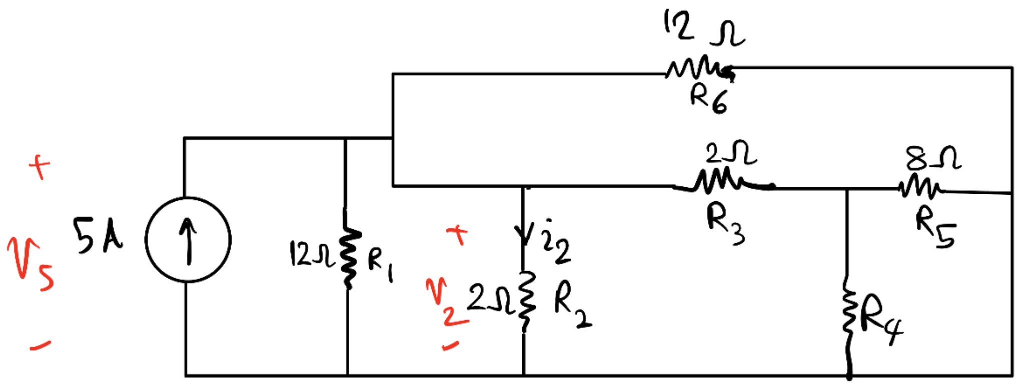
$$\text{then} \quad 3 = \frac{6\,R_{eq2}}{6 + R_{eq2}} \Rightarrow \boxed{R_{eq2} = 6\,\Omega}$$

$$\text{then} \quad 6 = 2 + 8 \parallel R_4 \Rightarrow \frac{8R_4}{8+R_4} = 4\,\Omega$$

$$\Rightarrow \boxed{R_4 = 8\,\Omega}$$

Sub-point (c)

To find the power delivered by the source we need the voltage across it. With reference to the schematic below



the solution proceeds as follows:

we can use the following expression:

$$P_s = -V_s \times I_s = -V_s \times 5A$$

looking at the circuit we have: $V_2 = V_s$

and $R_2 = \frac{V_2}{i_2}$ then $V_2 = 2 \times 3 = 6V$

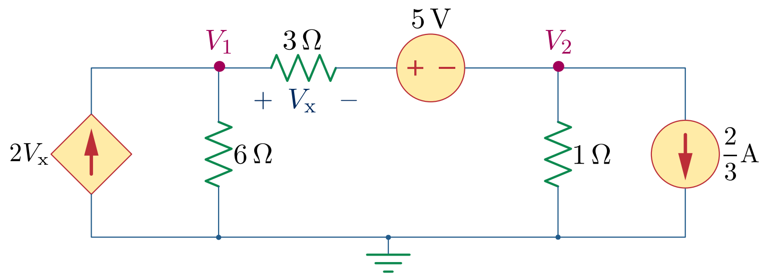
then $P_s = -6 \times 5 = \boxed{-30W}$

Note that, just like in sub-point (a), this result again assumes the passive sign convention for the source. If an active sign were used (as it may be suggested by "delivered") the sign would be plus.

- Take a new double-sheet -

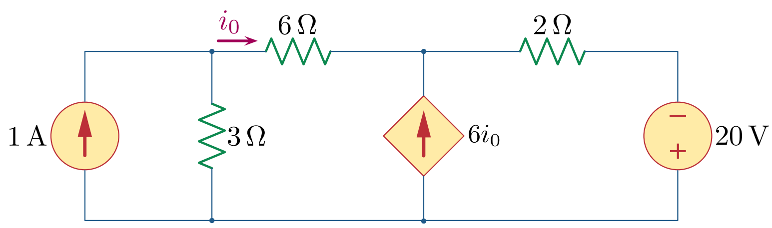
Exercise 3

Consider the circuit in the figure below:



a) Calculate voltages V_1 and V_2 using the nodal analysis. (6 points)

Now consider the second circuit in the figure below:



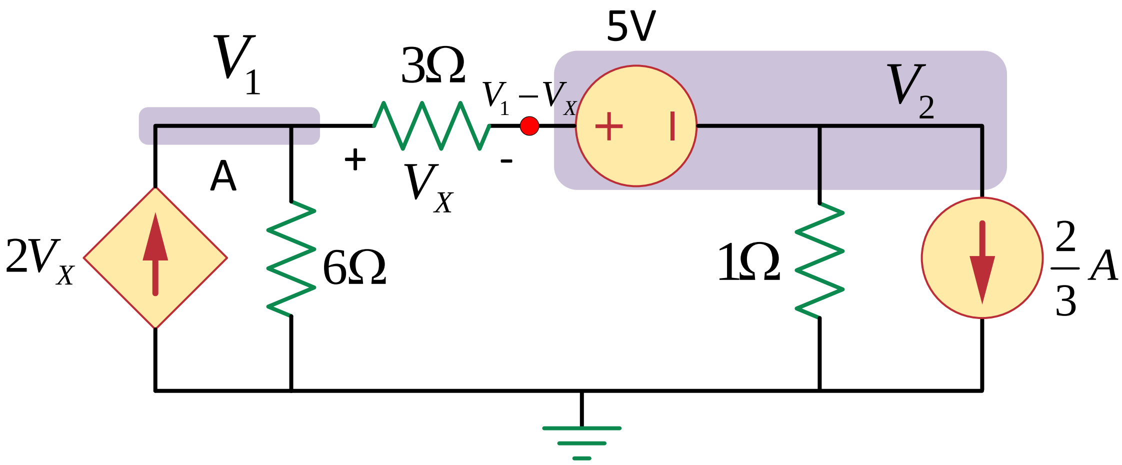
b) Calculate current i_0 using mesh analysis. (4 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

We write KCL at node A



$$\text{KCL (A)}: \ \frac{V_1}{6} + \frac{V_X}{3} - 2V_X = 0$$

$$V_1 = 10V_X$$

We now write KCL for super node shown on the right side of the circuit:

$$\text{KCL}: \ \frac{V_2}{1} - \frac{V_X}{3} + \frac{2}{3} = 0$$

$$3V_2 - V_X = -2$$

Also we know that:

$$V_1 - V_X - V_2 = 5$$

Combining the first equation with the third equation, and using the second equation we have:

$$9V_X - V_2 = 5$$

$$3V_2 - V_X = -2$$

Multiplying the first equation above by 3 and adding to the second equation:

$$27V_X - 3V_2 = 15$$

$$3V_2 - V_X = -2$$

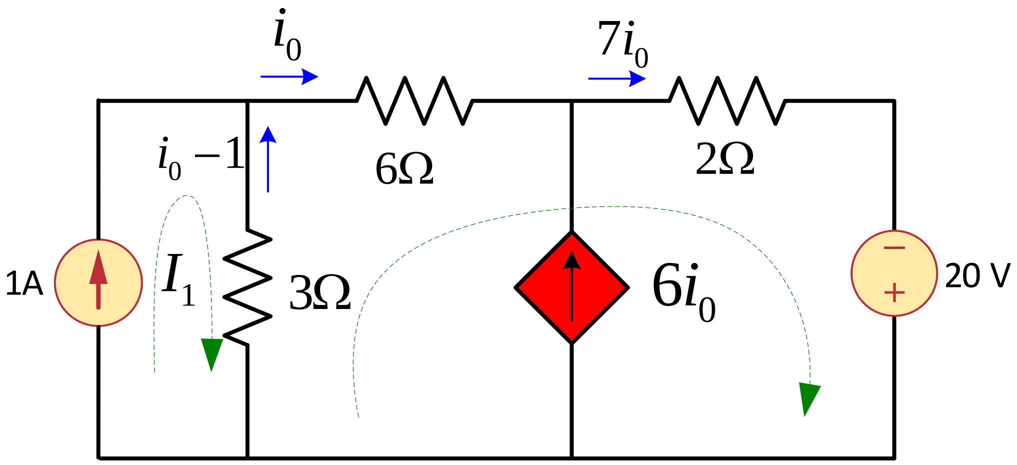
$$26V_X = 13 \rightarrow V_X = 0.5\ V$$

$$V_1 = 5\ V$$

$$V_2 = -0.5\ V$$

Sub-point (b)

By simple inspection we can see that the current of the left mesh is 1A and therefore current of 3 Ohm resistor is $i_0 - 1$ A. We can write the super mesh on the right side.



$$\text{KVL: } 3(i_0 - 1) + 6i_0 + 14i_0 - 20 = 0$$

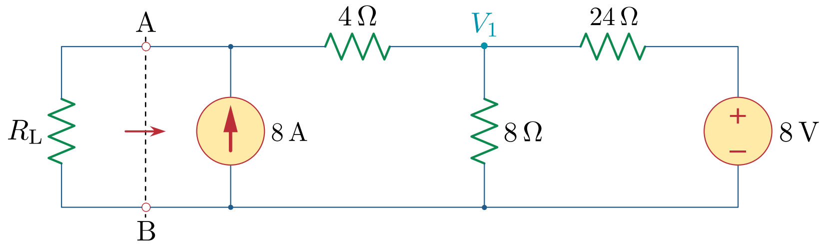
$$23i_0 = 23$$

$$i_0 = 1\ A$$

- Take a new double-sheet -

Exercise 4

Consider the circuit in the figure below:



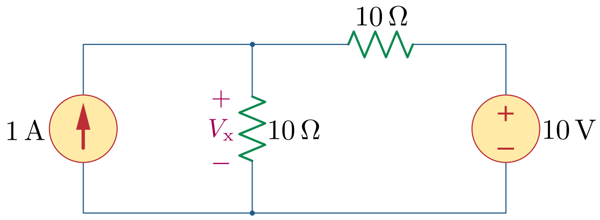
a) Using the Thévenin theorem, calculate the value of the Thévenin resistance R_{Th} to the right of the terminals A-B. (3 points)

b) Using the Thévenin theorem, calculate the value of the Thévenin voltage V_{Th} to the right of the terminals A-B. (3 points)

Hint: Use nodal analysis applied at the node indicated by V_1.

c) Draw the Thévenin equivalent circuit. (1 point)

Now consider the new circuit in the figure below:



d) Use superposition to determine the voltage V_x. (3 points)

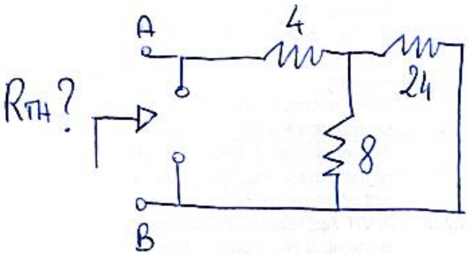
Hint: Recall that when applying superposition you have to analyse two simplified circuits and then sum their results.

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

The circuit only contains independent sources, hence it lends itself to turning all sources off (passivation) and calculating the equivalent resistance. The resulting circuit is shown below:

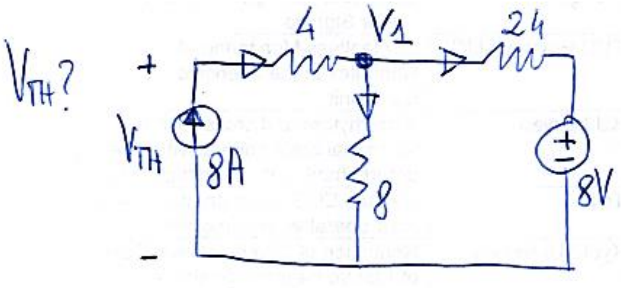


the thought for resistance being then:

$$R_{TH} = (24 // 8) + 4 = \frac{24 \cdot 8}{24+8} + 4 = \underline{10\Omega}$$

Sub-point (b)

The circuit at the right of the terminals A-B is shown below:



For calculating the Thévenin voltage V_{Th} we apply KCL at the node 1 → the relevant equations are given below

$$\frac{V_{TH} - V_1}{4} = \frac{V_1}{8} + \frac{V_1 - 8}{24}$$

its solution being:

$$8 = \frac{V_1}{8} + \frac{V_1 - 8}{24} \quad \Big| \quad 24 \cdot 8 = 3V_1 + V_1 - 8 \quad \Big| \quad 4V_1 = 200 \quad V_1 = 50V$$

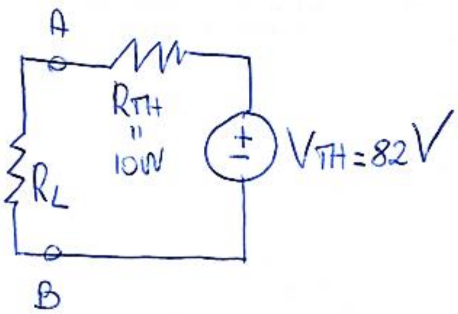
The final step is applying KVL on the left loop, this yielding:

$$\frac{V_{TH} - V_1}{4} = 8 \longrightarrow V_{TH} - V_1 = 32 \quad \underline{V_{TH} = 82V}$$

which it the final result.

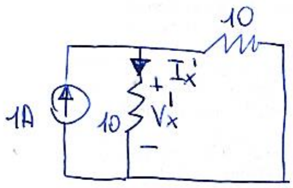
Sub-point (c)

The Thévenin equivalent is:



Sub-point (d)

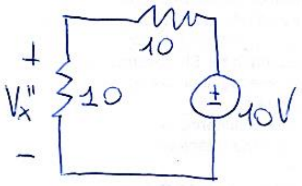
Applying superposition requires two steps. Turning off the 10V voltage source results in the circuit:



$$I_x' = \frac{1 \cdot 10}{10+10} = 0.5A$$

$$V_x' = 10 \cdot I_x' = 5V$$

while turning off the 1A current source results in the circuit:



$$V_x'' = \frac{10 \cdot 10}{10+10} = 5V$$

The final result is obtained by adding the two partial results:

$$V_x = V_x' + V_x'' = \underline{10V}$$