End-of-term Exam

EE1C21 “Linear Circuits B”

Place:
Date:
Time:

- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The exam grade is obtained by dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **When requested, fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly and avoid messy solutions. Should errors occur in your solution, cross the erroneous part out and give clear indications on where the correct solution resumes.
- For this exam you are allowed to use:
    i. a simple calculator – programmable and graphing calculators are explicitly prohibited;
    ii. a handwritten, double-sided A4 sheet with formulas.
- The text of this exam is offered only in English. Inasmuch as possible, instructors will assist you with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

Laplace transform pairs.*

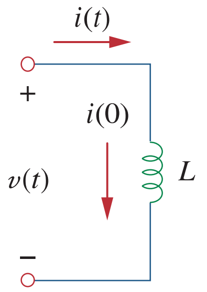
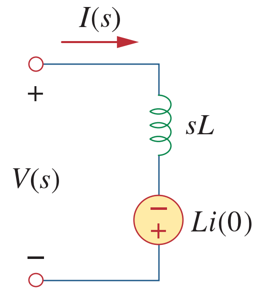
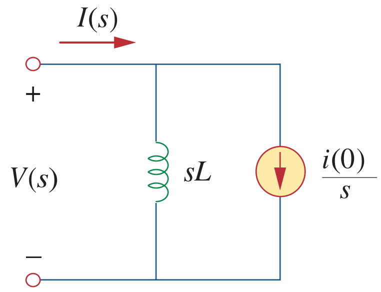
$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t+\theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
$\cos(\omega t+\theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Laplace-domain equivalent circuits for inductances and capacitances

<table>
<tr><th>Time-domain circuit</th><th>Thévenin-type equivalent</th><th>Norton-type equivalent</th></tr>
<tr><td>
</td><td>
</td><td>
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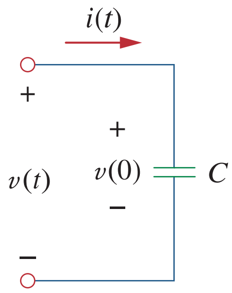
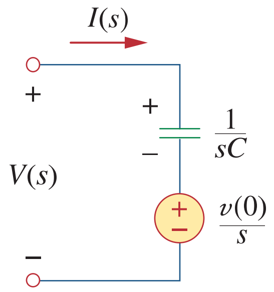
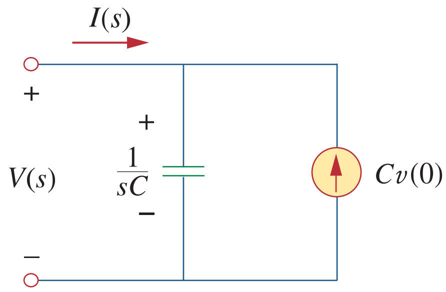
<tr><td>
</td><td>
</td><td>
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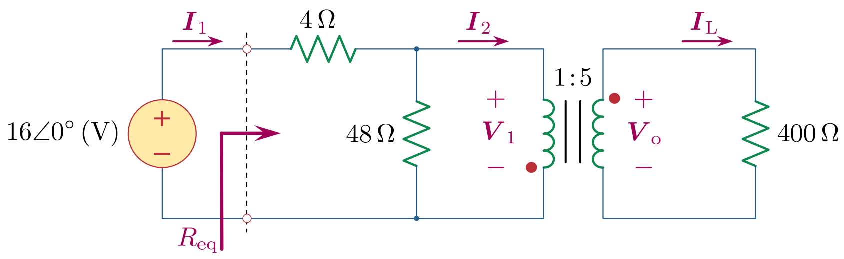
Initial-conditions voltage/current values: $v(0) = v(0^-)$ and $i(0) = i(0^-)$.

- Take a new double-sheet -

Exercise 1

Consider the circuit in the figure below, in which the transformer ***is ideal***:

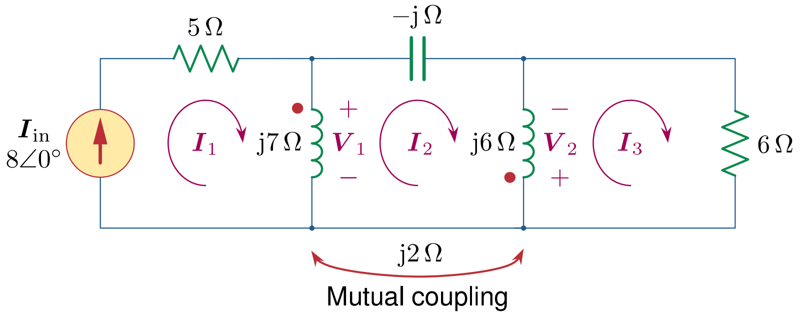


a) Calculate the equivalent resistance R_{eq}. (1 point)

b) Calculate the circuit quantities $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_L$ and $\boldsymbol{V}_o$. (2 points)

c) Calculate the power dissipated in the $400\,\Omega$ resistor. (1 point)

Now, consider the circuit with magnetically coupled inductances in the figure below:



d) Redraw the circuit. Include the controlled voltage sources corresponding to the induced voltages, by also specifying their polarity and value. (2 points)

e) Express the phasor voltages $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ in terms of the phasor currents $\boldsymbol{I}_1$, $\boldsymbol{I}_2$ and $\boldsymbol{I}_3$. (2 points)

f) Give the mesh equations for $\boldsymbol{I}_2$ and $\boldsymbol{I}_3$ and determine the values of both $\boldsymbol{I}_2$ and $\boldsymbol{I}_3$. (2 points)
(*Hint:* Please note that $\boldsymbol{I}_1$ is already given!)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

Exercise 2

Determine the Laplace transform of the following functions:

a) $f(t) = \left(3t^2 + 2t + 1\right) e^{-5t}\, u(t)$ (2 points)

b) $g(t) = e^{-2t} \cos(t-4)\, u(t-4)$ (3 points)

Now, find the inverse Laplace transform of the following *s*-domain transfer functions:

c) $F(s) = \dfrac{s+4}{(s+3)s}$ (2 points)

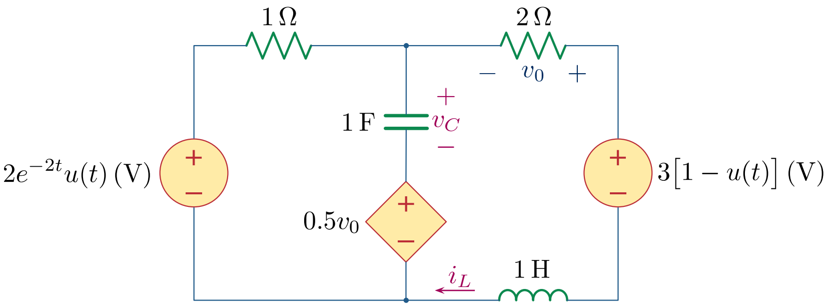
d) $G(s) = \dfrac{s+5}{s^2+2s+17}$ (3 points)

Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

Exercise 3

Consider the circuit in the figure below:



a) Calculate $v_C(0^+)$ and $i_L(0^+)$. (2 points)

b) Redraw the circuit in the Laplace-domain. (2 points)

c) Calculate the Laplace-domain current $\boldsymbol{I}_L(s)$. (4 points)
(*Hint:* Try applying mesh analysis.)

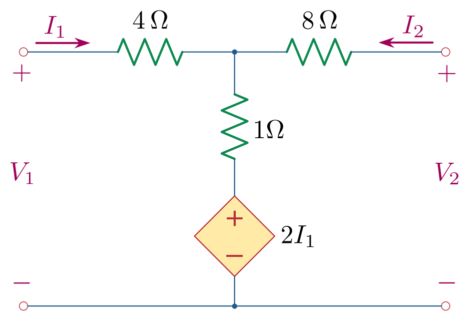
d) Calculate the time-domain current $i_L(t)$ for $t > 0$. (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

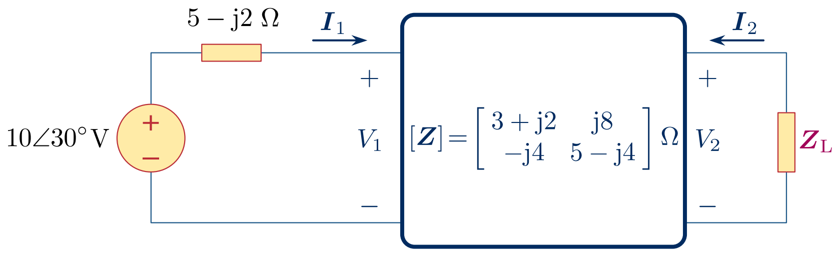
Exercise 4

Consider the circuit in the figure below:



a) Calculate the $\boldsymbol{Z}$–parameters $\boldsymbol{z}_{11}$, $\boldsymbol{z}_{12}$, $\boldsymbol{z}_{21}$ and $\boldsymbol{z}_{22}$. (5 points)

Now consider the new circuit in the figure below:



b) Redraw the circuit with all independent sources pasivised (set to zero) and the load replaced by a current source with the value $\boldsymbol{I}_{test}$. (1 point)

c) Using the new circuit, find the Thévenin impedance seen at the load's terminals. (3 points)

d) Calculate $\boldsymbol{Z}_L$ for maximum power transfer. (1 point)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.