End-of-term Exam

EE1C21 “Linear Circuits B”

Place: Drebbelweg Exam Hall 2
Date: 03-02-2023
Time: 9:00 – 11:00

- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The exam grade is obtained by dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **When requested, fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly and avoid messy solutions. Should errors occur in your solution, cross the erroneous part out and give clear indications on where the correct solution resumes.
- For this exam you are allowed to use:
    1. a simple calculator – programmable and graphic calculators are explicitly prohibited;
    2. a handwritten, double-sided A4 sheet with formulas.
- The text of this exam is offered only in English. Inasmuch as possible, instructors will assist you with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t+\theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
$\cos(\omega t+\theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Laplace-domain equivalent circuits for inductances and capacitances

<table>
<tr><th>Time-domain circuit</th><th>Thévenin-type equivalent</th><th>Norton-type equivalent</th></tr>
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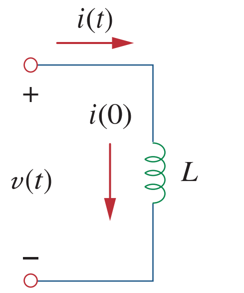
<td>
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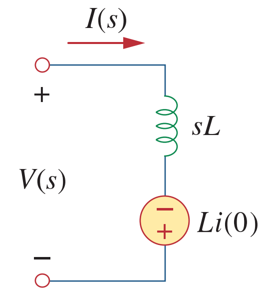
<td>
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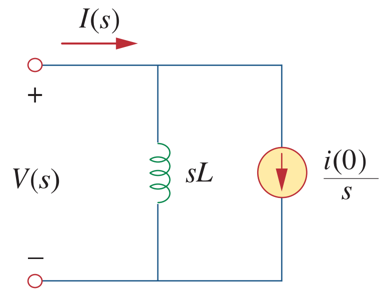
<td>
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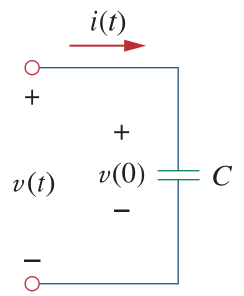
<td>
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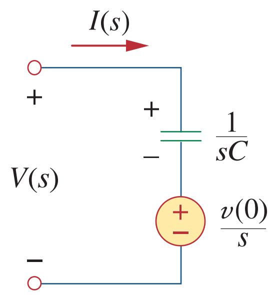
<td>
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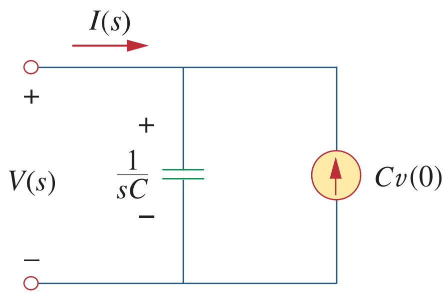
<td>
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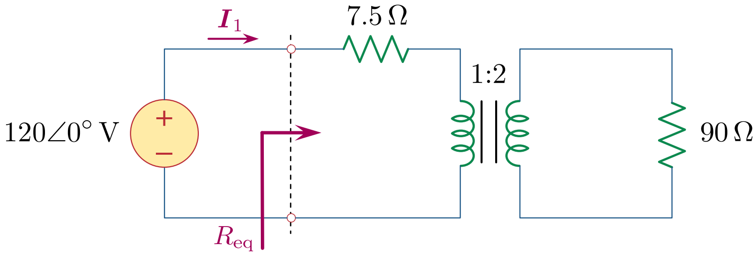
Initial-conditions voltage/current values: $v(0) = v(0^-)$ and $i(0) = i(0^-)$.

- Take a new double-sheet -

Exercise 1

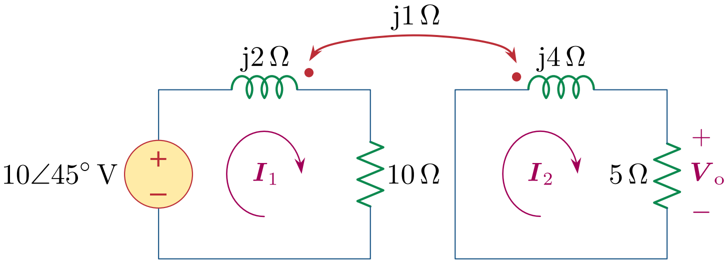
Consider the circuit with an ideal transformer in the figure below:



a) Calculate the equivalent resistance R_{eq}. (1 point)

b) Calculate the current $\boldsymbol{I}_1$. (1 point)

Now consider the circuit with magnetically coupled inductors in the figure below:

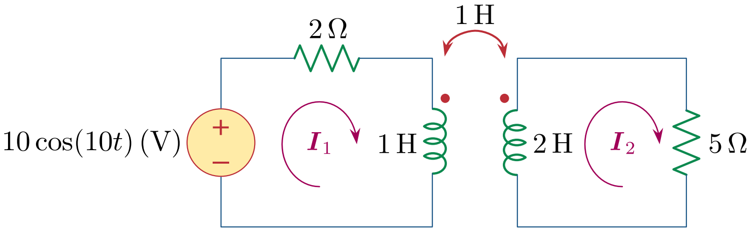


c) Calculate the coupling coefficient k assuming $\omega = 1$ rad/s. (1 point)

d) Redraw the equivalent circuit where the magnetically coupled inductors are replaced by normal inductors and dependent voltage sources. (1 point)
Hint: Remember the dot convention for the polarity of the voltage sources.

e) Write the mesh equations for the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ and then calculate the value of $\boldsymbol{V}_o$ using the value of $\boldsymbol{I}_2$. (2 points)

Now consider the new circuit with magnetically coupled inductors in the figure below:



f) Redraw the equivalent circuit where the magnetically coupled inductors are replaced by normal inductors and dependent voltage sources. (1 point)

g) Write the mesh equations for the currents $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ and then calculate their expression in the time domain. (2 points)

h) Calculate the energy stored in the magnetically coupled inductors at time $t = 1$s. (1 point)

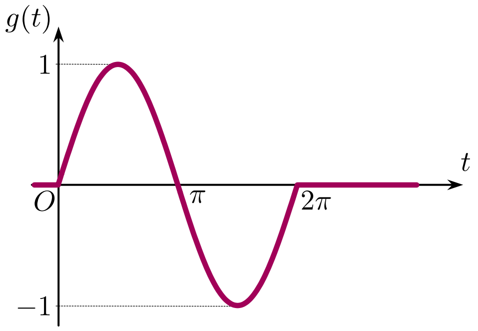
Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

Exercise 2

a) Calculate the Laplace transform $F(s)$ of the function $f(t) = e^{-2t}u(t-\tau)$. (2 points)

b) Calculate the Laplace transform $G(s)$ of the function shown in the plot below. (2 points)



c) Calculate the inverse Laplace transform $f(t)$ of the s-domain transfer function $F(s) = \dfrac{s+1}{s+3}$. Use the final value theorem to verify your answer. (2 points)

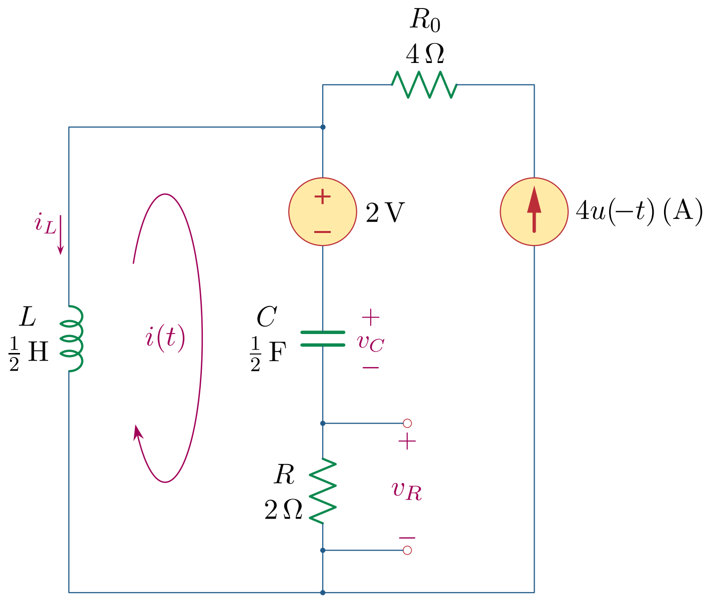
d) Calculate the inverse Laplace transform $g(t)$ of the s-domain transfer function $G(s) = \dfrac{3s+9}{(s^2+s+1)(s+2)}$. (4 points)

Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

Exercise 3

Consider the circuit in the figure below:



a) Calculate $v_C(0-)$ and $i_L(0-)$ in the time-domain. (1 point)

b) Redraw the circuit in the s-domain, by also accounting for the initial states and their values. (3 points)

c) Obtain the expression of the current $\boldsymbol{I}(s)$ in the s-domain that corresponds to the mesh current $i(t)$. (3 points)

d) What type of damping we have in this circuit? (1 point)

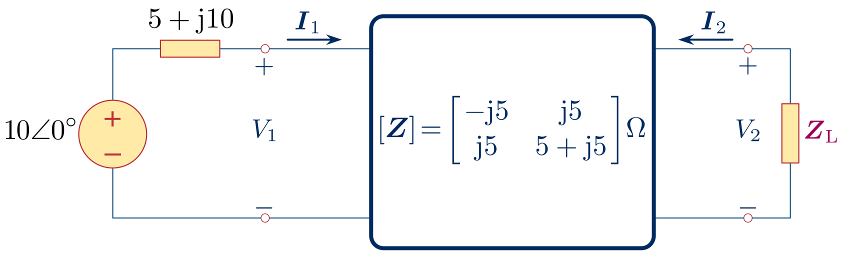
e) Calculate the inverse Laplace transform $i(t)$ and obtain the resistor voltage $v_R(t)$ in the time-domain for $t > 0$. (2 points)

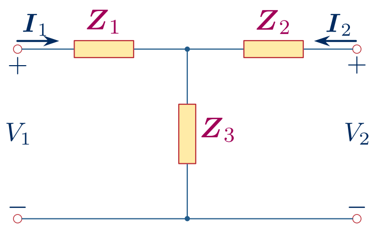
Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

- Take a new double-sheet -

Exercise 4

Consider the circuit at the left-hand side, in the figure below:



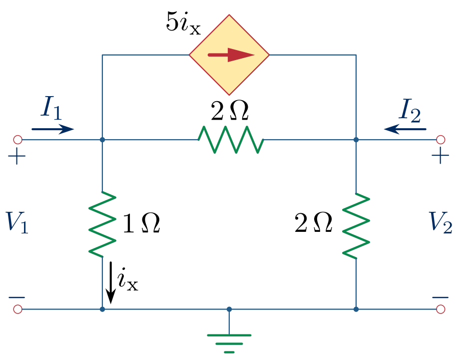


a) Calculate the impedances $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$ and $\boldsymbol{Z}_3$ of its T equivalent circuit shown at the right-hand side of the figure. (1 points)

b) Calculate the Thévenin impedance $\boldsymbol{Z}_{\text{Th}}$ and voltage $\boldsymbol{V}_{\text{Th}}$ of the Thévenin equivalent seen at the terminals of the load $\boldsymbol{Z}_{\text{L}}$. (3 points, 1.5 points each)
Hint: You may consider replacing the two-port by its T equivalent circuit derived at sub-point (a).

c) Calculate the maximum power transferred to the load $\boldsymbol{Z}_{\text{L}}$, under conjugate matching conditions. (1 point)

Now consider the new circuit in the figure below:



d) Calculate the $\boldsymbol{Y}$–parameters $\boldsymbol{y}_{11}$, $\boldsymbol{y}_{12}$, $\boldsymbol{y}_{21}$ and $\boldsymbol{y}_{22}$. (5 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.