End-of-term exam

EE1C2 “Linear Circuits B”

Place: Drebbelweg Hall 2
Date: 02-02-2024
Time: 9:00 – 11:00

- This exam consists of 4 exercises.
- Each exercise accounts for **10 points**; the total number of points to be obtained is **40**. The exam grade is obtained by dividing the total number of points by 4, rescaling linearly the result to the 1-10 scale and rounding off to 1 decimal.
- **Each exercise must be solved on a separate double-sheet.** Writing more solutions on the same sheet may result in only one of the solutions being graded!
- Indicate your name and study number on **each** submitted sheet. **You must hand in (blank) signed sheets even for the exercises that you do not handle.**
- Students benefitting of the “Extra Time” (ET) rule are entitled to a 20 minutes extension of their exam provided they produce the relevant supporting document.
- Should any question not be completely clear, you are allowed to ask the instructors in the exam hall; the answer will be confined to rephrasing the text of the exercise such that to make it more intelligible.
- Should a part of an exercise depend on a previous result, mistakes made at a previous step will only be penalised once.
- Give your solution as completely as possible and never state numerical results without indicating how you derived them. **Simply stating numerical results will yield no points.**
- **When requested, fill in the measure units for all calculated quantities.** This holds for intermediate results but definitely for the final ones.
- Write clearly and avoid messy solutions. Should errors occur in your solution, cross the erroneous part out and give clear indications on where the correct solution resumes.
- For this exam you are allowed to use:
    i. a simple calculator – programmable and graphic calculators are explicitly prohibited;
    ii. a handwritten, double-sided A4 sheet with formulas.
- The text of this exam is offered only in English. Inasmuch as possible, instructors will assist you with the Dutch translation of formulations that you may have difficulties to understand.

The Linear Circuits team wishes you a lot of success!

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s + a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s + a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s + a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s + a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s + a}{(s + a)^2 + \omega^2}$

Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t - a)u(t - a)$	$e^{-as} F(s)$
Frequency shift	$e^{-at} f(t)$	$F(s + a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s} F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
Time periodicity	$f(t) = f(t + nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$
Initial value	$f(0)$	$\lim_{s \to \infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s \to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s) F_2(s)$

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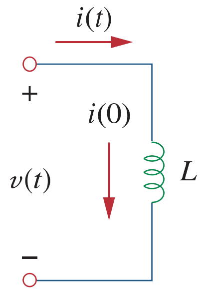
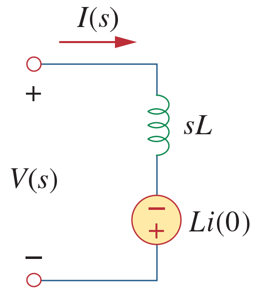
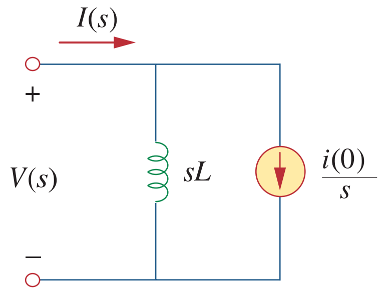
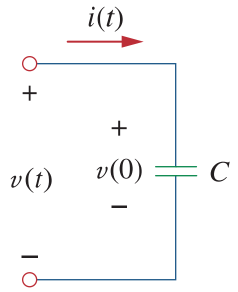
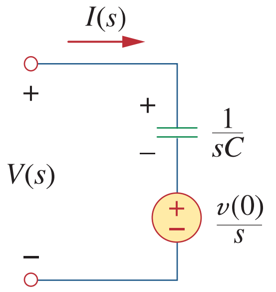
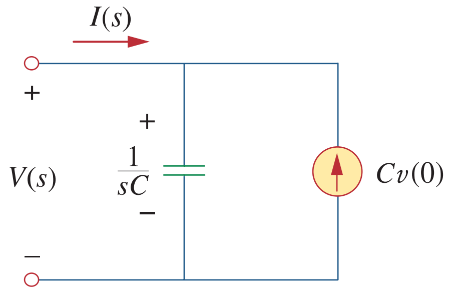
Laplace-domain equivalent circuits for inductances and capacitances

Time-domain circuit	Thévenin-type equivalent	Norton-type equivalent
		
		

Initial-conditions voltage/current values: $v(0) = v(0^-)$ and $i(0) = i(0^-)$.

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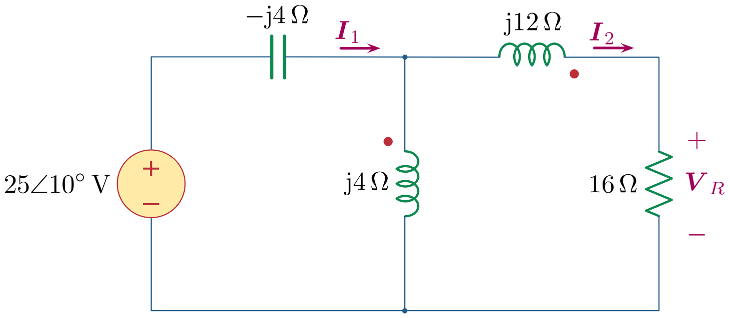
- Take a new double-sheet -

Exercise 1

A team from your company has designed the circuit in the figure below under the assumption that the inductors are decoupled. They ask you to help them analysing the circuit.



a) Calculate the values I_1, I_2 and V_R. (3 points)

After building the circuit you found that, due to close proximity, the inductors are coupled with a coupling impedance of j12Ω, the dotted terminals being indicated in the figure.

b) Calculate the new values I_1, I_2 and V_R, by accounting for the mutual coupling between the inductors. (6 points)

c) Calculate the average power absorbed by the resistance when the inductors are uncoupled, and when they are coupled. (1 point)

Indicate the measure units for all calculated quantities (when applicable). Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

Using mesh analysis, we obtain the following equation for left mesh:

(j4-j4) I_1-j4I_2=25∠10° ⟺ -j4I_2=25∠10°

Thus: I_2=j 25/4∠10° =6.25 ∠ 100° A

From the right mesh, we obtain:

-j4I_1+16(1+j) I_2=0 ⟺I_1=16/4j (1+j) I_2

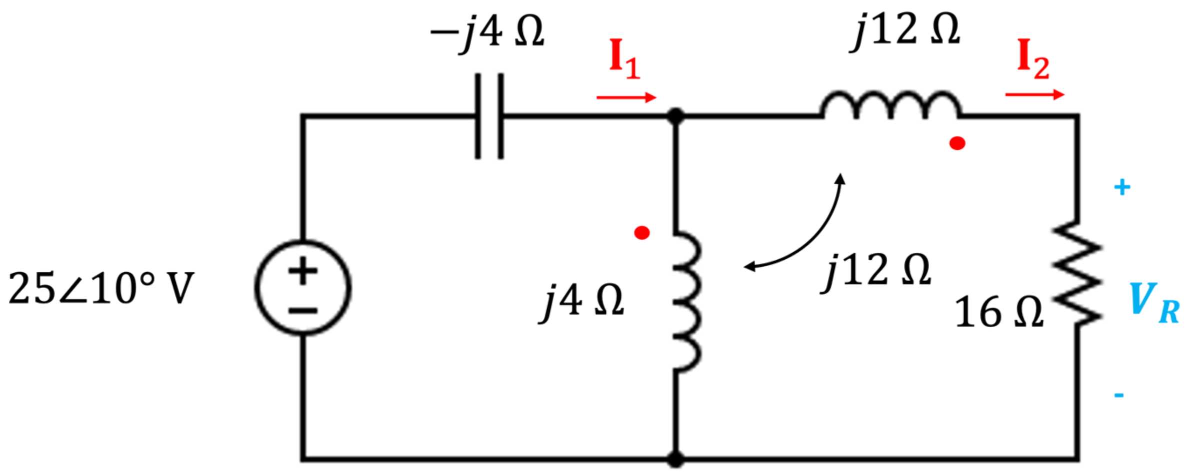
Thus: I_1=(1+j) 25∠10° = 35.4∠ 55° A

Using Ohm’s law, we obtain:

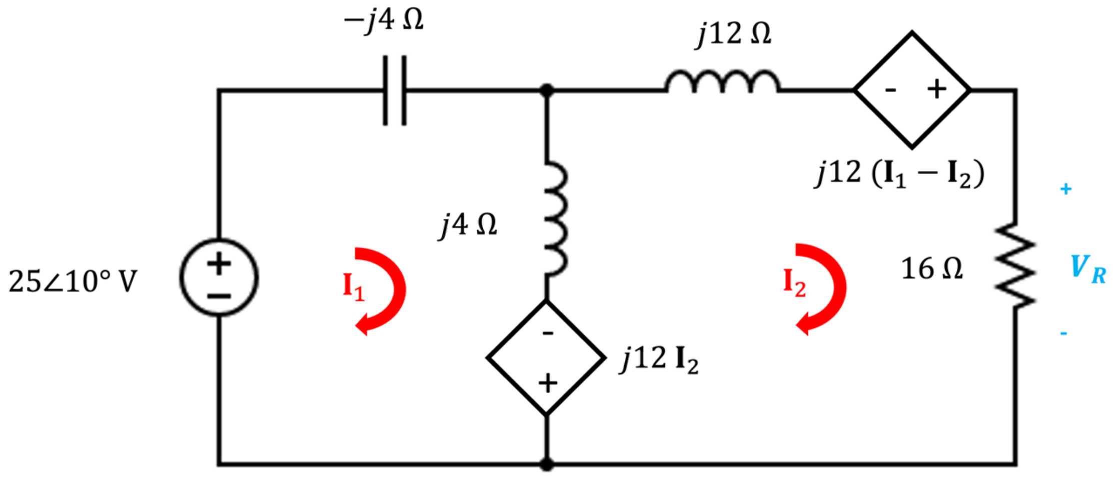
V_R=RI_2=16×6.25∠100° = 100∠100° V

Sub-point (b)

Since the two coils are coupled, we have the following circuit:



We can build an equivalent circuit with dependent sources being careful to use the dot convention:



Analyzing this circuit for the left mesh, we obtain:

$$(\mathrm{j}4-\mathrm{j}4)\mathrm{I}_1-\mathrm{j}4\mathrm{I}_2-\mathrm{j}12\mathrm{I}_2=25\angle 10^\circ \quad \Leftrightarrow \quad -\mathrm{j}16\mathrm{I}_2=25\angle 10^\circ$$

Thus: $\mathrm{I}_2 = j\,\frac{25}{16}\angle 10^\circ \;= 1.6\angle\,100^\circ$ A

From the right mesh, we obtain:

$$-(\mathrm{j}12+\mathrm{j}4)\mathrm{I}_1+(16+12\mathrm{j}+16\mathrm{j})\mathrm{I}_2=0 \;\Leftrightarrow I_1=\frac{1}{4j}(4+10j)I_2$$

Thus: $\mathrm{I}_1 = (4+10j)\,\frac{25}{64}\angle 10^\circ \;= 4.2\angle\,78.2^\circ$ A

Using Ohm's law, we obtain:

$$\mathrm{V}_R = R\mathrm{I}_2 = 16\times\frac{25}{16}\angle 100^\circ \;=\; 25\angle 100^\circ \text{ V}$$

Sub-point (c)

The average power definition is given by:

$$\mathrm{P}=\frac{1}{2}V_m I_m\cos(\theta_v-\theta_i)$$

Thus:

$$\mathrm{P}_{no\_coupling}=\frac{1}{2}100\times 6.25\cos(100-100)=312.5\text{ W}$$

and:

$$\mathrm{P}_{coupling}=\frac{1}{2}25\times 1.6\cos(100-100)=19.5\text{ W}$$

- Take a new double-sheet -

Exercise 2

a) Find the Laplace transform of the function: (5 points)

$$f(t) = te^{-3t}\sin(4t)$$

Hint: Compute the Laplace transform F(s) using given table of Laplace transform pairs and any necessary properties of the Laplace transform.

b) Find the inverse Laplace transform of the expression: (5 points)

$$\frac{3s-2}{s^2+4s+5}$$

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

$$\frac{8(s+3)}{((s+3)^2+16)^2}$$

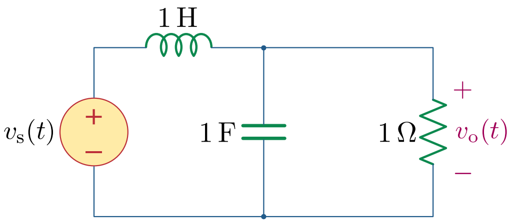
Sub-point (b)

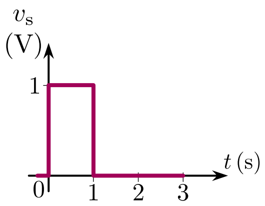
$$3 \cdot e^{-2t} \cdot \left(\cos(t) - \frac{8\sin(t)}{3}\right)$$

- Take a new double-sheet -

Exercise 3

Consider the circuit in the figure below:

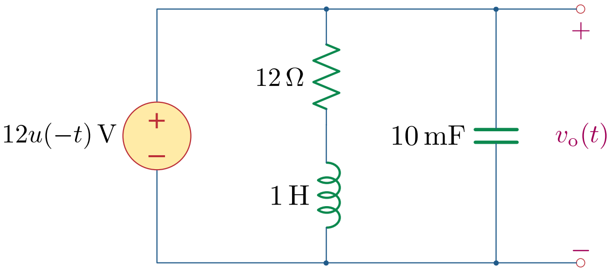




a) Convert the circuit and the source to the s domain. (2 points)

b) Calculate the value of $v_o(t)$ for $t > 0$. (6 points)

Now consider the new circuit in the figure below:



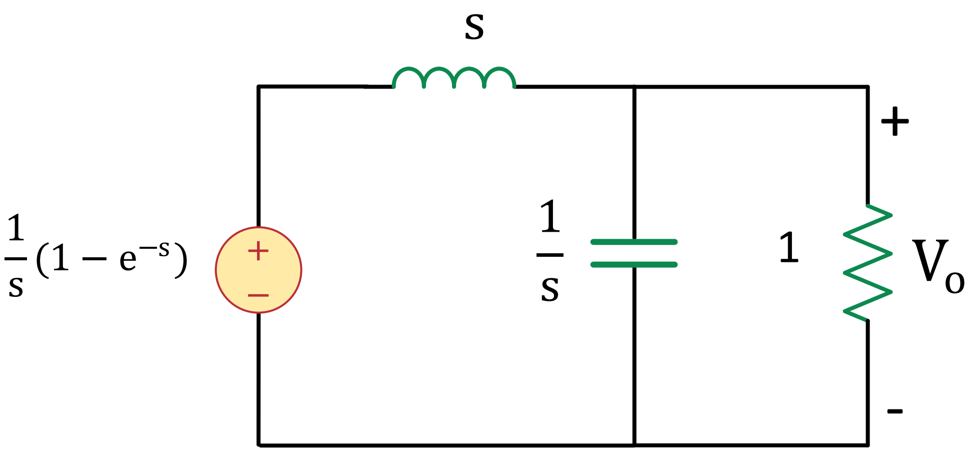
c) Convert the circuit and the source to the s domain. (2 points)

Indicate the measure units for all calculated quantities. Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

The circuit in s domain is shown below:



Sub-point (b)

$$V_0 = \frac{\frac{1\times\frac{1}{s}}{1+\frac{1}{s}}}{\frac{1\times\frac{1}{s}}{1+\frac{1}{s}}+s}\left(\frac{1}{s}(1-e^{-s})\right) = \frac{\frac{1}{s+1}}{\frac{1+s+s^2}{s+1}}\left(\frac{1}{s}(1-e^{-s})\right) = \frac{1-e^{-s}}{s(1+s+s^2)}$$

$$V_0 = \frac{1}{s(1+s+s^2)} - \frac{1e^{-s}}{s(1+s+s^2)}$$

The second part is the same as the first part only has a shift in time domain.

$$\frac{1}{s(1+s+s^2)} = \frac{A}{s} + \frac{Cs+D}{1+s+s^2}$$

$$A = s\frac{1}{s(1+s+s^2)}\bigg|_{s\,=\,0} = 1$$

$$V_0 = \frac{1}{s} + \frac{Cs+D}{1+s+s^2} = \frac{(1+s+s^2)+s(Cs+D)}{s(s+1)(1+s+s^2)}$$

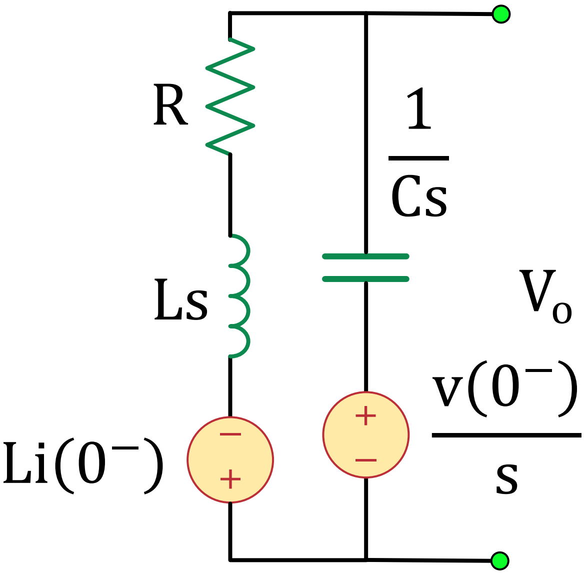
The coefficient of s^2 must be zero, hence C=-1 and The coefficient of s must be zero, hence $D = -1$

$$\frac{1}{s} - \frac{s+1}{1+s+s^2} = \frac{1}{s} - \frac{(s+0.5)+0.5}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{s} - \frac{(s+0.5)}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{(s+0.5)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$v_0(t) = \left(1 - e^{-0.5t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)u(t)$$
$$-\left(1 - e^{-0.5(t-1)}\cos\left(\frac{\sqrt{3}}{2}(t-1)\right) - \frac{1}{\sqrt{3}}e^{-0.5(t-1)}\sin\left(\frac{\sqrt{3}}{2}(t-1)\right)\right)u(t-1)$$

Sub-point (c)

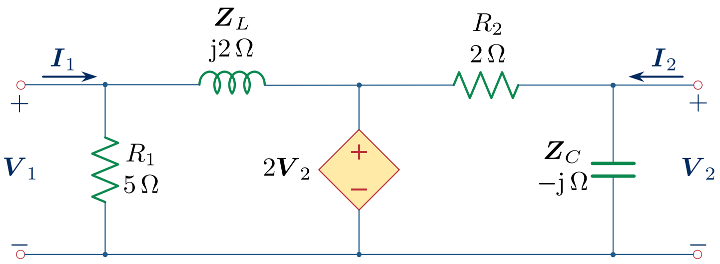
The initial current of the inductor is 1A and initial voltage of the capacitor is 12 V. The equivalent circuit in the Laplace domain is:



- Take a new double-sheet -

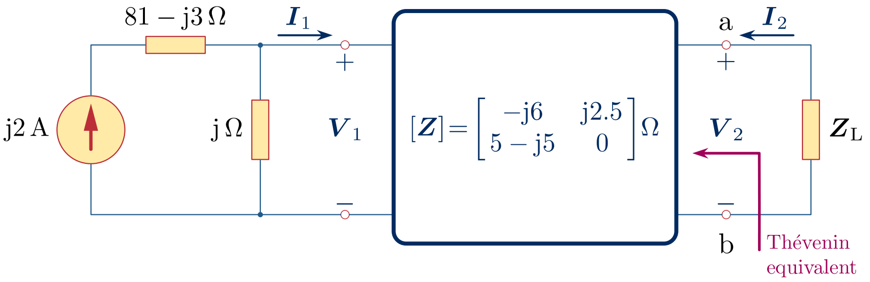
Exercise 4

Consider the phasor-domain circuit in the figure below:



a) Represent the two-port network above using admittance parameters in the phasor domain. (5 points)

Now, consider the new phasor-domain circuit in the figure below:



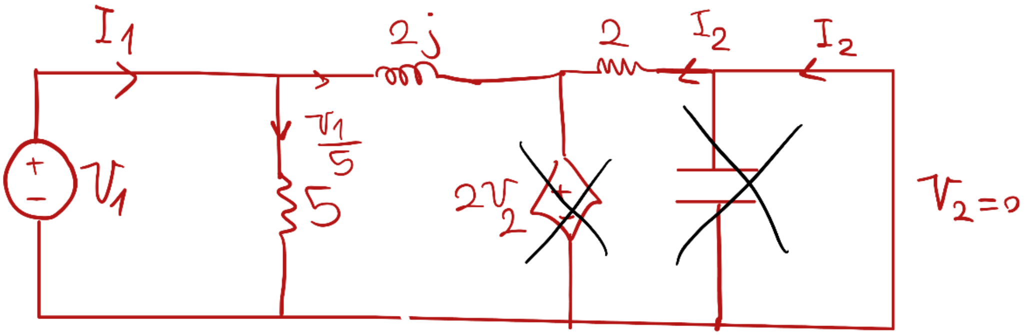
b) Find the equivalent Thévenin circuit seen at the $a - b$ terminals in this circuit. (5 points)

Indicate the measure units for all calculated quantities (when applicable). Show all steps in your reasoning and never give numerical results without justification.

Solution

Sub-point (a)

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \quad \text{and} \quad y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

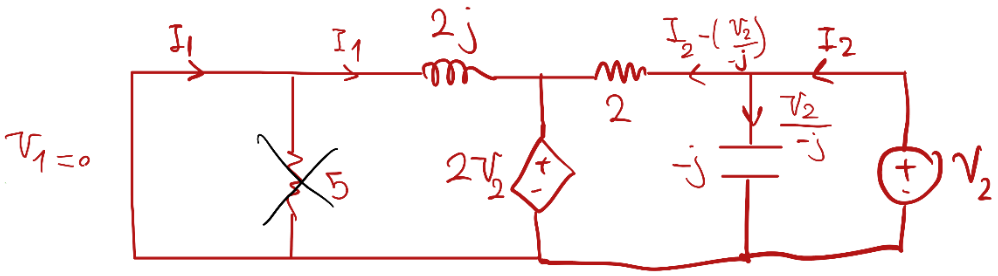


$$\left.\begin{aligned} &\text{kvl } I_1: \quad -V_1 + 2j\left(I_1 - \frac{V_1}{5}\right) + 2\cancel{V_2}^{0} = 0 \\ &\text{kvl } I_2: \quad 2I_2 + 2\cancel{V_2}^{0} = 0 \Rightarrow I_2 = 0 \end{aligned}\right\}$$

$$\rightarrow \quad 2j\,I_1 = V_1\left(1 + \frac{2j}{5}\right) \Rightarrow \frac{I_1}{V_1} = Y_{11} = \frac{1}{2j} + \frac{1}{5} = \boxed{0.2 - j0.5}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} \qquad I_2 = 0 \Longrightarrow Y_{21} = 0$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \qquad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$



kvl I_1: $2jI_1+2V_2=0 \rightarrow \frac{I_1}{V_2}=\frac{-2}{2j}=Y_{12}=\boxed{j}$

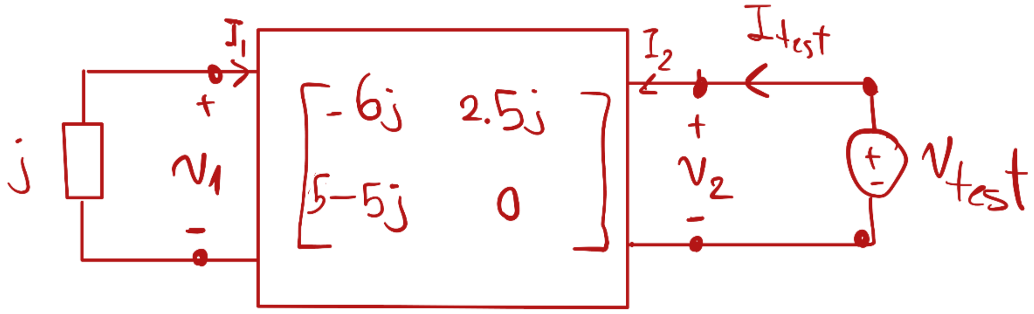
kvl I_2: $-V_2+2\left(I_2-\frac{V_2}{-j}\right)+2V_2=0$

$\rightarrow 2I_2=V_2\left(-1+\frac{2}{-j}\right) \Rightarrow \frac{I_2}{V_2}=Y_{22}=\frac{\frac{2}{-j}-1}{2}=-0.5+j$

$$Y=\begin{bmatrix} 0.2-0.5j & j \\ 0 & -0.5+j \end{bmatrix}$$

Sub-point (b)

$Z_{th} \rightarrow$ independent sources off



$$Z_{th} = \frac{V_{test}}{I_{test}} \qquad V_{test} = V_2$$

Z matrix :
$$\begin{cases} V_1 = -6j\, I_1 + 2.5j\, I_2 & ① \\ V_2 = [5-5j]\, I_1 & ② \end{cases}$$

KVL I_1: $I_1(\ j\ ) + V_1 = 0$

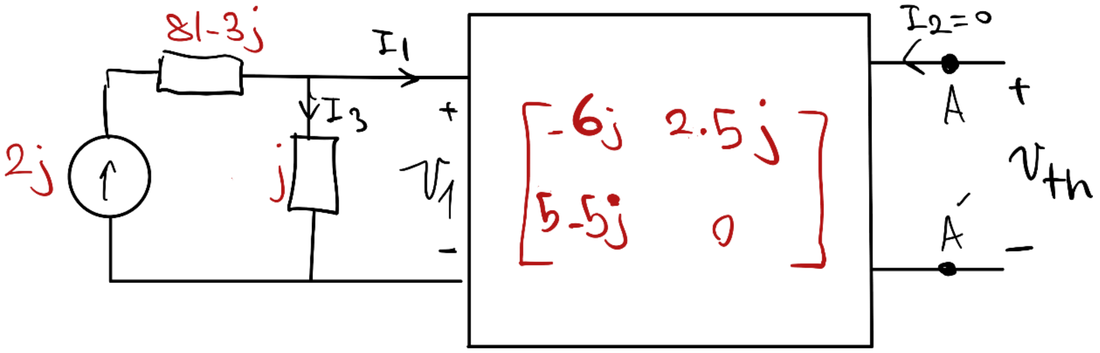
$$\rightarrow V_1 = -j I_1 \qquad \xrightarrow{①}$$

$$I_1(-j + 6j) = 2.5j\, I_2 \Rightarrow I_1 = \frac{I_2}{2}$$

$$\xrightarrow{②} \quad V_2 = \frac{5-5j}{2} I_2 = 2.5(1-j)\, I_2$$

$$\rightarrow \frac{V_2}{I_2} = \boxed{2.5-2.5j = Z_{th}}$$

V_{th} : Value of voltage at the terminal



$I_2 = 0$

$I_3 = \frac{V_1}{j}$ ①

$V_1 = -6jI_1 + 2.5j\,\cancel{I_2}^{0} = -6jI_1$ ②

$V_2 = [5-5j]I_1$ ③

$$I_g = I_1 + I_3 \xrightarrow[②]{①} 2j = I_1\left(1 + \frac{-6j}{j}\right)$$

$$\Rightarrow 2j = I_1(1-6) \Rightarrow I_1 = \frac{2j}{-5}$$

$$\Rightarrow I_1 = -0.4j$$

$$\rightarrow V_{th} = V_2 = (5-5j)(-0.4j)$$

$$\boxed{= -2-2j}$$