Exam 2 - EE1M1 Calculus (30/01/2024 09:00 - 11:00)

Fill in your personal information and
write down your answers for the eight short-answer questions and
write down all your steps for the five open question and
hand in when finished.

You are allowed to use:

- Pen, pencils and scrap paper;
- A simple calculator;
- The formula sheet;

Short-answer questions

An explanation is not required for the short-answer questions. Only the answer matters. The maximum points per question is indicated in the margin.

Clearly write the answer in the box. You do not need to fully simplify your answers.

1. (2 + 2 pt) Let $f(x, y) = 3x^3 - 2xy^2$.

   a. Compute the gradient ∇f of f.

   b. For which direction $\mathbf{u}$ does $D_{\mathbf{u}}f(1, 2)$ reach its minimal value?

   a. $\nabla f =$

   b. $\mathbf{u} =$

2. (4 pt) Find all critical points of the function $f(x, y) = x^2 - 2x + 3y - y^3$ and classify them as local maxima, local minima or saddle points.

   Local maxima:

   Local minima:

   Saddle points:

3. (4 pt) Reverse the order of integration for $\int_0^4 \int_{1-\frac{x}{2}}^1 f(x, y)\, dy\, dx$ and give the resulting integral.

   $\int_0^4 \int_{1-\frac{x}{2}}^1 f(x, y)\, dy\, dx =$

4. (4 pt) Is the vector field $\mathbf{F}(x, y, z) = \langle ze^{xz}, 1 + z, xe^{xz} + y\rangle$ conservative? If it is conservative, give a potential function ϕ.

   ☐ Conservative ☐ Not conservative

   If conservative, $\phi =$

5. (6 pt) Let $\mathcal{D}$ be the region in $\mathbb{R}^2$ bounded by the circle $(x - 2)^2 + y^2 = 4$, the x-axis and the line $y = x$. A charge density $q(x, y)$ is distributed over this region. Express the net total charge Q on $\mathcal{D}$ as a double integral in polar coordinates.

   $Q =$

6. (6 pt) Let $\mathcal{E}$ be the solid region in $\mathbb{R}^3$ which is given by the part with $y \leq 0$ of the region in between the cone $z = \sqrt{3x^2+3y^2}$ and the plane $z = 2$. Express the integral $\iiint_{\mathcal{E}} \sqrt{x^2+y^2+z^2}\, dV$ as a triple integral in spherical coordinates.
You do not need to evaluate the integral!

$$\iiint_{\mathcal{E}} \sqrt{x^2+y^2+z^2}\, dV =$$

Open questions

The next questions need to be worked out completely, every answer needs to be reasoned. Make the exercises in the box. If necessary, there is extra space at the back of the exam. If you use this extra space, clearly indicate the numbering of the questions there AND write in the regular answer box that you use the extra space. The maximum points per question is indicated in the margin.

7. (6 pt) Let $\mathcal{C}$ be the triangle with vertices $(0,0)$, $(-1,2)$ and $(-1,-2)$ with clockwise orientation. Evaluate $\oint_{\mathcal{C}} (2y - \cos(x))\, dx + \left(4x + e^{2y}\right)\, dy$.

8. (8 pt) Let $\mathcal{D}$ be the region in $\mathbb{R}^2$ bounded in between the parabola $y = x^2$ and the line $y = 2$. Find the absolute minimum and absolute maximum of the function $f(x, y) = 6x^4 + y^3 - 6y^2 + 9y$ on $\mathcal{D}$ and the points at which these values occur.

9. (6 pt) Consider the coordinate transformation $\left\{\begin{array}{lcl} u & = & 2x+y \\ v & = & x-2y \end{array}\right.$. Let $\mathcal{D}$ be the region enclosed by the lines $y=3-2x, y=4-2x, y=\frac{x}{2}-1$ and $y=\frac{x}{2}-2$. Express and evaluate the integral $\displaystyle\iint_{\mathcal{D}}(3x-y)^2\,dA$ using uv-coordinates. If needed, you may use that $\dfrac{\partial(x,y)}{\partial(u,v)}=\dfrac{1}{\dfrac{\partial(u,v)}{\partial(x,y)}}$.

10. (6 pt) Let $\mathcal{C}$ be the curve in $\mathbb{R}^3$ that starts at the point $(1,0,0)$ and spirals once around the cylinder $x^2+z^2=1$ along a circular helix in counterclockwise direction when viewed from the negative y-axis, and ends at the point $(1,2\pi,0)$. Consider the vector field $\mathbf{F} = \langle 2xyz, x^2z+1+x, x^2y \rangle$. Evaluate $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

$$\textbf{Grade} = \frac{\text{obtained points}}{6} + 1$$

THE END

Extra space 1 (Clearly indicate which question this extra space relates to).

Extra space 2 (Clearly indicate which question this extra space relates to).