Practice Exam with Grasple part - EE1M1 Calculus

You are allowed to use:

- Pen, pencils and scrap paper.

The formula sheet can be found on the next page.

Formula sheet

Some trigonometric formulae

$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$
$\cos(2\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$

Some limits

$$\lim_{x\to\infty} \frac{x^p}{e^x} = 0$$
$$\lim_{x\to\infty} \left(1 + \frac{a}{x}\right)^x = e^a$$
$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0 \quad (p > 0)$$

Some integrals

$$\int \frac{\mathrm{d}\,x}{\sin(x)} = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\cos(x)} = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{1 + x^2} = \arctan(x) + C$$
$$\int \frac{\mathrm{d}\,x}{1 - x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{1 - x^2}} = \arcsin(x) + C = -\arccos(x) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2 + 1}} = \ln\big(x + \sqrt{x^2+1}\big) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2 - 1}} = \ln\big|x + \sqrt{x^2-1}\big| + C$$
$$\int \sqrt{1 + x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\big(x + \sqrt{1+x^2}\big) + C$$
$$\int \sqrt{1 - x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) + C$$
$$\int_0^{\frac{\pi}{2}} \sin^n x\ \mathrm{d}\,x = \begin{cases} \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\cdots\frac{3}{4}\frac{1}{2}\frac{\pi}{2} & \text{if } n \text{ even and } n \geq 2 \\ \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\cdots\frac{4}{5}\frac{2}{3} & \text{if } n \text{ odd and } n \geq 3 \end{cases}$$

Grasple questions
The first seven questions should be made on Grasple, using the following link.

Make sure to follow the input format for each exercise. Although only the final answer is graded, partial credits can be awarded for certain partially correct answers.

Open questions
The next questions need to be worked out completely, every answer needs to be reasoned.

8. Consider a differentiable function $f(x, y, z)$. Suppose the directional derivative of f at the point $(3, 1, 4)$ in the direction of $\mathbf{u} = \langle 1, 2, 2\rangle$ is equal to 3 and suppose that the directional derivative of f at $(3, 1, 4)$ is minimal in the direction of $\mathbf{v} = \langle -2, 0, -1\rangle$. Find $\nabla f(3, 1, 4)$.

9. Let $\mathcal{E}$ be the solid region inside the sphere $x^2 + y^2 + (z-1)^2 = 1$ that has $y \le 0$. Assume that a charge is distributed over $\mathcal{E}$ with charge density function $\sigma(x, y, z) = (x^2 + y^2 + z^2)^{\frac{3}{2}}$ Find the net total charge on $\mathcal{E}$.

10. Evaluate the iterated integral

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{0}\int_{x^2+y^2}^{2x^2+2y^2}(z-x^2-y^2)^{\frac{3}{2}}\,dz\,dy\,dx$$

11. Let $\mathbf{F}(x, y)$ be a conservative vector field on $\mathbb{R}^2$. In addition, consider the vector field $\mathbf{G}(x, y) = \langle xy, -xy \rangle$. Let $\mathcal{D}$ be the region bounded in between the lines $x + y = 1$, $x + y = 2$, $x - y = 3$ and $x - y = 5$ and let $\partial\mathcal{D}$ denote the boundary of $\mathcal{D}$ with clockwise orientation.
Evaluate $\int_{\partial\mathcal{D}} (\mathbf{F}(x, y) + \mathbf{G}(x, y)) \cdot d\mathbf{r}$.

THE END