TUDelft

Exam 1 - EE1M1 Calculus (12/12/2023 09:00 - 11:00)

Fill in your personal information and
write down your answers for the eight short-answer questions and
write down all your steps for the five open question and
hand in when finished.

You are allowed to use:
- Pen, pencils and scrap paper;
- A simple calculator;
- The formula sheet;

Short-answer questions

An explanation is not required for the short-answer questions. Only the answer matters. The maximum points per question is indicated in the margin.

Clearly write the answer in the box. You do not need to fully simplify your answers.

1. (3 pt) Let $f(x, y) = 2x^2y - 2y^3$. Find an equation of the tangent plane to the graph of f at the point $(2, -1, -6)$.

2. (4 pt) Evaluate the limit

$$\lim_{x \to -\infty} \sqrt{4x^2 - 3x + 1} + 2x + 1.$$

3. (4 pt) Simplify $\tan(\arccos(x))$ into an expression that does not involve trigonometric functions or their inverses.

4. (4 pt) Consider the curve defined by $y - e^y = x^2y$ in $\mathbb{R}^2$. Determine $\dfrac{dy}{dx}$.

5. (4 pt) Let $z(s, t) = f(u(s, t), v(s, t))$, where f, u, and v are differentiable with the following values given:

Function	Value at $(1, 4)$	Value at $(7, 9)$
f	3	9
f_u	8	3
f_v	-2	1
u	7	4
u_s	6	7
u_t	-1	-3
v	9	1
v_s	4	9
v_t	8	4

So, for example, $u(1, 4) = 7$.
Find $z_s(1, 4)$.

6. (6 pt) Evaluate the integral

$$\int \sin(x) \cos(x) \ln(\cos(x)) \, dx.$$

7. (4 pt) Find the angle (in radians) between the vectors $\mathbf{u} = \langle 2, 0, -2 \rangle$ and $\mathbf{v} = \langle 2, 1, -1 \rangle$.

8. (4 pt) Evaluate the limit

$$\lim_{x \to 0^+} x^{2x/(\ln(x)\sin(x))}.$$

Open questions

The next questions need to be worked out completely, every answer needs to be reasoned. Make the exercises in the box. If necessary, there is extra space at the back of the exam. If you use this extra space, clearly indicate the numbering of the questions there AND write in the regular answer box that you use the extra space. The maximum points per question is indicated in the margin.

9. (6 pt) Consider the complex polynomial $p(z) = z^3 - 4z^2 + 14z - 20$. One of the roots of $p(z)$ is $z = 2$. Find the roots of this polynomial (repeated according to their multiplicity).
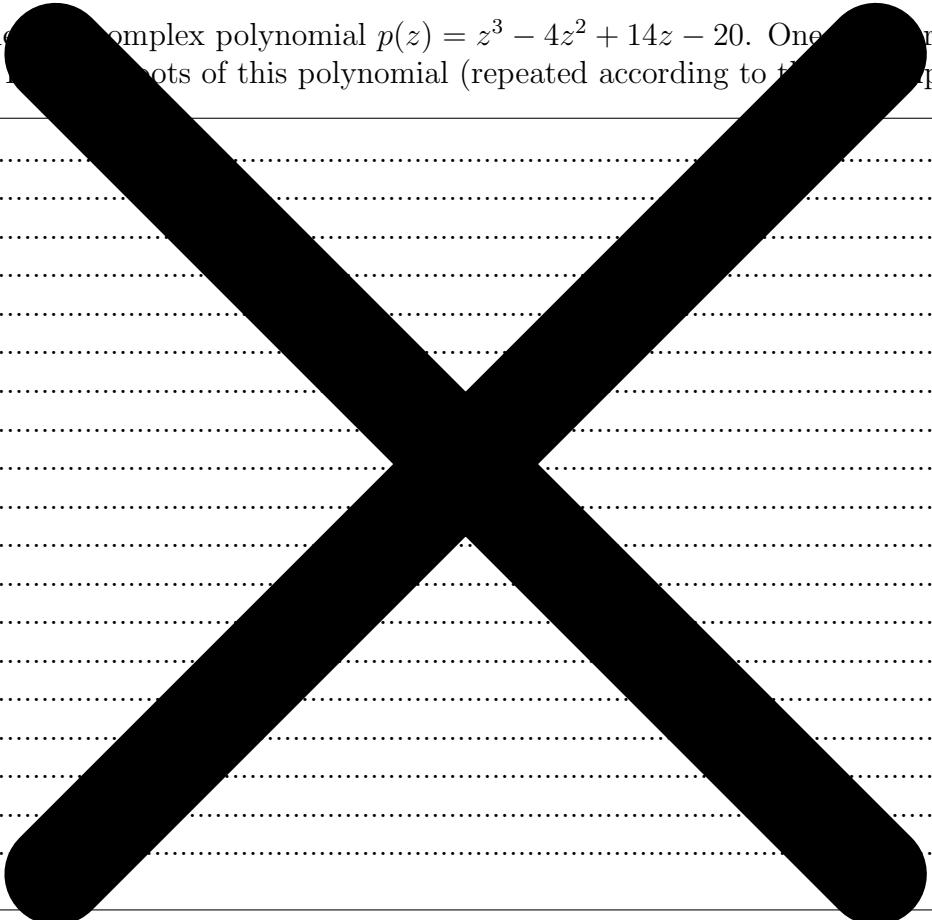
10. (6 pt) Let A be the point $(1, 0, 1)$ and let V be the plane through the points $(2, 1, 3)$, $(3, 1, 3)$ and $(2, 0, 2)$. Find the distance between A and V.

11. (8 pt) Evaluate the integral $\displaystyle\int \frac{x^2 + 3x + 4}{(x-1)(x^2 + 2x + 5)}\, dx$.

12. (6 pt) Determine whether the following integral is convergent or divergent, and in case of convergence compute the integral

$$\int_2^\infty \frac{x^2 + 3}{\sqrt{x^6 - 3x^2}}\, dx.$$

13. (4pt) Show that the equation $x^3 - 8x + 6 = 0$ has at least one solution between 0 and 1.

$$\mathbf{Grade} = \frac{\text{obtained points}}{7} + 1$$

THE END

Extra space 1 (Clearly indicate which question this extra space relates to).

Extra space 2 (Clearly indicate which question this extra space relates to).