Exam 1 - EE1M1 Calculus (11/12/2024 09:00 - 11:00)

Fill in your personal information and answer the eight questions in Grasple and write down all your steps for the four open question and submit in when finished.

Student number: _____

You are allowed to use:

- Pen, pencils and scrap paper.

How to start your exam:

1. Go to the **Brightspace** page of this course.
2. From 15 minutes before the scheduled start time:
   Click on the **link to Grasple** in the new **exam announcement.**
3. Log in using your **TU Delft credentials** (a.k.a. NetID).
   You should be able to do this without a password manager!
4. Open the provided test and click the **Launch Schoolyear browser** button.
5. Schoolyear will start, after which you again log in using your **TU Delft credentials**.
6. *From 5 minutes past the scheduled start time:*
   Start, perform and **submit the test** shown.

When you are finished with the exam, please follow these steps:

1. **Submit the exam** in Grasple.
2. **Close** Schoolyear
3. Gather all your items and move **quietly** to the exit of the exam room
4. Hand in **all scrap paper** at the exit
5. Hand in this **exam sheet**..
6. Leave the exam room.

Formula sheet

Some trigonometric formulae

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

Some limits

$$\lim_{x\to\infty} \frac{x^p}{e^x} = 0$$
$$\lim_{x\to\infty} \left(1 + \frac{a}{x}\right)^x = e^a$$
$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0 \quad (p > 0)$$

Some integrals

$$\int \frac{\mathrm{d}\,x}{\sin(x)} = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\cos(x)} = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{1+x^2} = \arctan(x) + C$$
$$\int \frac{\mathrm{d}\,x}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{1-x^2}} = \arcsin(x) + C = -\arccos(x) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2+1}} = \ln\left(x + \sqrt{x^2+1}\right) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2-1}} = \ln\left|x + \sqrt{x^2-1}\right| + C$$
$$\int \sqrt{1+x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\left(x + \sqrt{1+x^2}\right) + C$$
$$\int \sqrt{1-x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) + C$$
$$\int_0^{\frac{\pi}{2}} \sin^n x\ \mathrm{d}\,x = \begin{cases} \dfrac{n-1}{n}\dfrac{n-3}{n-2}\dfrac{n-5}{n-4}\cdots\dfrac{3}{4}\dfrac{1}{2}\dfrac{\pi}{2} & \text{if } n \text{ even and } n \geq 2 \\ \dfrac{n-1}{n}\dfrac{n-3}{n-2}\dfrac{n-5}{n-4}\cdots\dfrac{4}{5}\dfrac{2}{3} & \text{if } n \text{ odd and } n \geq 3 \end{cases}$$

Short-Answer questions (total: 27 points)

1. (3 pt) Simplify the trigonometric expression $\sin(\arccos(3x))$. Your final answer may not contain any (inverse) trigonometric functions.

2. (3 pt) The curve C is implicitly described by the equation
$e^{-3y} = 2x^3y^2 + 2x^2 + 2y^2 - 7$
and passes through the point $(2, 0)$. Find the tangent line to C at $(2, 0)$.

3. (4 pt) Evaluate the following integral
$\int 2x^3\sqrt{x^2+4}\,dx$
Use a capital C as a constant of integration.

4. (4 pt) Consider the function $f(x, y)$, where $x = x(s, t)$ and $y = y(s, t)$. You are given the following values for the functions and their partial derivatives:

$f(2,3) = -3$, $\frac{\partial f}{\partial x}(2,3) = 4$, $\frac{\partial f}{\partial y}(2,3) = 2$, $f(1,2) = -2$, $\frac{\partial f}{\partial x}(1,2) = -2$, $\frac{\partial f}{\partial y}(1,2) = 2$,

$x(2,3) = 1$, $\frac{\partial x}{\partial s}(2,3) = 1$, $\frac{\partial x}{\partial t}(2,3) = -1$, $x(1,2) = 2$, $\frac{\partial x}{\partial s}(1,2) = 1$, $\frac{\partial x}{\partial t}(1,2) = -1$,

$y(2,3) = 2$, $\frac{\partial y}{\partial s}(2,3) = 1$, $\frac{\partial y}{\partial t}(2,3) = -1$, $y(1,2) = 3$, $\frac{\partial y}{\partial s}(1,2) = -1$, $\frac{\partial y}{\partial t}(1,2) = 1$.

We have $h(s, t) = f(x(s, t), y(s, t))$. Find $\frac{\partial h}{\partial s}(1, 2)$.

Since 2025-2026 this exercise is part of partial exam 2, instead of partial exam 1

5. (2 pt) Match the contour diagrams to the correct function. The options for the functions are $\frac{x+2y}{1+4x^2+y^2}$, $\frac{x+2y}{1+x^2+4y^2}$, $\frac{x-2y}{1+4x^2+y^2}$, $\frac{x-2y}{1+x^2+4y^2}$.

Diagram 1:

Diagram 2:

Since 2025-2026 this exercise is part of partial exam 2, instead of partial exam 1

Diagram 1

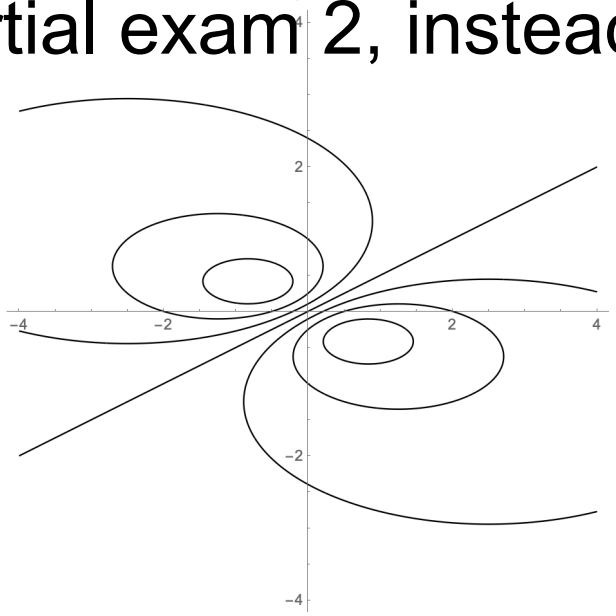
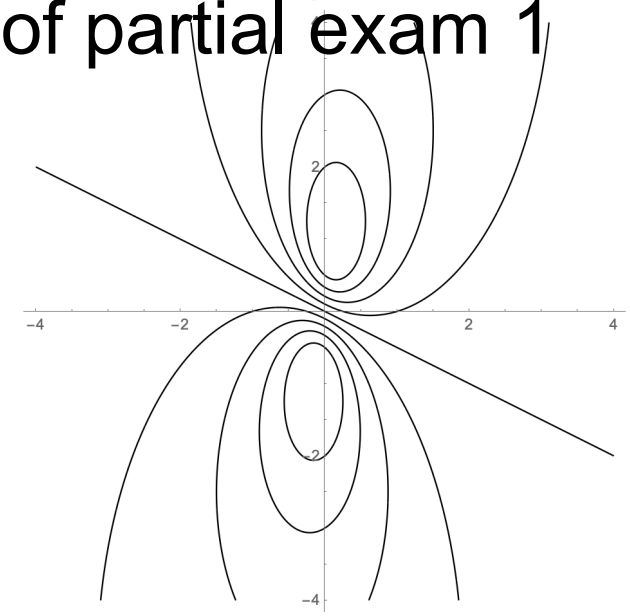


Diagram 2



6. (3 pt) Given are the vectors
$$\mathbf{a} = \begin{bmatrix} \frac{5\sqrt{3}}{2} \\ 0 \\ -\frac{5}{2} \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{2} \\ -\frac{\sqrt{3}}{4} \end{bmatrix}.$$
Find the angle θ between the vectors $\mathbf{a}$ and $\mathbf{b}$.

7. (4 pt) Given the function $f(x,y) = 1 + \sqrt{x^2y - 28}$. Find the linearization $L(x,y)$ of the function at the point $(-4,4)$ and use this linearization to approximate the function value $f(-4.5, 3.9)$.

$L(x,y) =$

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$f(-4.5, 3.9) \approx$

8. (4 pt) Consider the lines L_1 and L_2 with the following parametrizations:
$$L_1 : \ \mathbf{r}_1(t) = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix} + t \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$$
$$L_2 : \ \mathbf{r}_2(t) = \begin{bmatrix} 8 \\ -2 \\ 8 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$
Find an equation of the plane that contains L_1 and is parallel to L_2. Write the equation in the form $ax + by + cz = d$.

Open questions (total: 25 points)
The next questions need to be worked out completely, every answer needs to be motivated. Write the solution in the box. If necessary, there is extra space at the end of the exam. If you use this extra space, clearly indicate the numbering of the questions there AND write in the regular answer box that you use the extra space. The maximum points per question is indicated in the margin.

9. (4 pt) Find an approximation for $e^{-\frac{1}{8}}$ using a Taylor polynomial of degree 3 of $f(x) = e^x$ around $x = 0$. What is the sharpest upper bound for the Lagrange remainder for this approximation?

10. (8 pt) Evaluate the integral

$$\int \frac{3x^4 - 6x^3 + 16x^2 - 20x + 22}{(x^2 - 2x + 1)(x^2 + 4)}\, dx$$

11. (4 + 4 pt) Evaluate the following limits.

(a). $\lim_{x\to 0}(1-x^2)^{\frac{1}{x^2}}$

(b). $\lim_{x\to\infty}\dfrac{\sin(3x^2)-\sqrt{4x^4+5x^3-3x^2}+2x^2}{2x+1}$

12. (5 pt) Determine whether the following integral is convergent or divergent. If the integral converges, you do not need to evaluate it.

$$\int_0^1 \frac{3-x^2}{2\sqrt{x}+x^3+\sin(x)}\,dx$$

Grade $= \dfrac{\text{obtained points}}{52} \cdot 9 + 1$

THE END

Extra space 1 (Clearly indicate which question this extra space relates to).

Extra space 2 (Clearly indicate which question this extra space relates to).