TU Delft

Practice Exam with Grasple part - EE1M1 Calculus

You are allowed to use:

- Pen, pencils and scrap paper.

The formula sheet can be found on the next page.

Formula sheet

Some trigonometric formulae

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

Some limits

$$\lim_{x\to\infty} \frac{x^p}{e^x} = 0$$
$$\lim_{x\to\infty} \left(1 + \frac{a}{x}\right)^x = e^a$$
$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0 \quad (p > 0)$$

Some integrals

$$\int \frac{\mathrm{d}\,x}{\sin(x)} = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\cos(x)} = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$$
$$\int \frac{\mathrm{d}\,x}{1+x^2} = \arctan(x) + C$$
$$\int \frac{\mathrm{d}\,x}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{1-x^2}} = \arcsin(x) + C = -\arccos(x) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2+1}} = \ln\left(x + \sqrt{x^2+1}\right) + C$$
$$\int \frac{\mathrm{d}\,x}{\sqrt{x^2-1}} = \ln\left|x + \sqrt{x^2-1}\right| + C$$
$$\int \sqrt{1+x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln\left(x + \sqrt{1+x^2}\right) + C$$
$$\int \sqrt{1-x^2}\ \mathrm{d}\,x = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) + C$$
$$\int_0^{\frac{\pi}{2}} \sin^n x\ \mathrm{d}\,x = \begin{cases} \dfrac{n-1}{n}\dfrac{n-3}{n-2}\dfrac{n-5}{n-4}\cdots\dfrac{3}{4}\dfrac{1}{2}\dfrac{\pi}{2} & \text{if } n \text{ even and } n \geq 2 \\ \dfrac{n-1}{n}\dfrac{n-3}{n-2}\dfrac{n-5}{n-4}\cdots\dfrac{4}{5}\dfrac{2}{3} & \text{if } n \text{ odd and } n \geq 3 \end{cases}$$

Grasple questions
The first seven questions should be made on Grasple, using the following link.

Make sure to follow the input format for each exercise. Although only the final answer is graded, partial credits can be awarded for certain partially correct answers.

Open questions
The next questions need to be worked out completely, every answer needs to be reasoned.

8. Evaluate the limit

$$\lim_{x\to 1}(2-x)^{\frac{1}{\sin(x-1)}}+e^{(x-1)\cos\left(\frac{1}{x-1}\right)-1}$$

9. Evaluate the integral

$$\int \frac{2x^3 + x^2 + x - 2}{(x-1)(1+x^2)}\, dx$$

10. Is there a function $f(x, y, z)$ that satisfies both $f_{xy}(x, y, z) = 6x^2y + 2xz$ and $f_{zx}(x, y, z) = 2xy + e^z + 2x$? If so, give an example of such a function. If not, show why this is impossible?

11. Determine whether the following integral is convergent or divergent. If the integral converges, you do not need to evaluate it.

$$\int_{-\infty}^{\infty} \frac{\arctan(x)}{1+x^2}\, dx.$$

12. Approximate $\sin(0.1)$ using a Taylor polynomial of degree 3 of $f(x) = \sin(x)$ around $x = 0$. What is the sharpest upper bound for the Lagarange remainder?

THE END