

EE1C1 “Linear Circuits A”

Week 1.2

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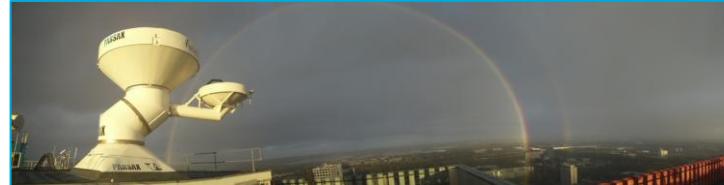
My group website: radar.tudelft.nl

Today

- **Recap:** AC-DC, resistors & resistance, independent & controlled sources
- **Week 1.2:**
 - Ohm's law, short circuit / open circuit, power dissipated in resistances
 - Kirchhoff's laws
 - Series and parallel circuits
 - Voltage division / current division (duality)
- **Summary and Next Week**

My own Intro

-Part of the MS3 (Microwave Sensing Signals & Systems) Group. Located on the **20th floor** of the EWI high-rise building, & above.



-What do we research? *We mostly work on Radar/Antennas/Electromagnetic.*

-What is that for? *Not only for defence/security, but for challenges in:*

- **Autonomous** vehicles and robots
- **Weather** forecasting for climate change, water management, agriculture
- **Contactless healthcare** support (e.g., monitoring vital signs).

-And Linear Circuits? *What you study in this course is fundamental to understand how a complex system like a radar works, and perhaps one day design one...*

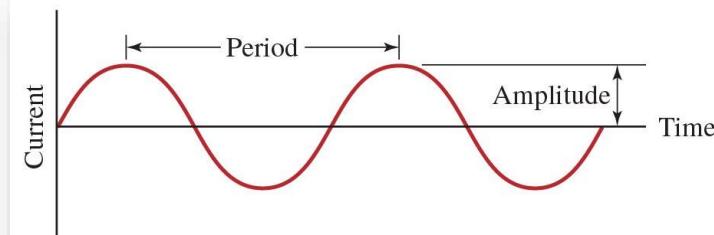
Recap

Electric Current: DC and AC*

- Flow of electric charge occurs in **one direction** and is **constant**.
- Frequency is zero (constant)
- Typically produced by batteries and direct current generator
- Flow of electric charge that periodically reverses
- Frequency – the number of cycles per second
- Created by generators at power plants



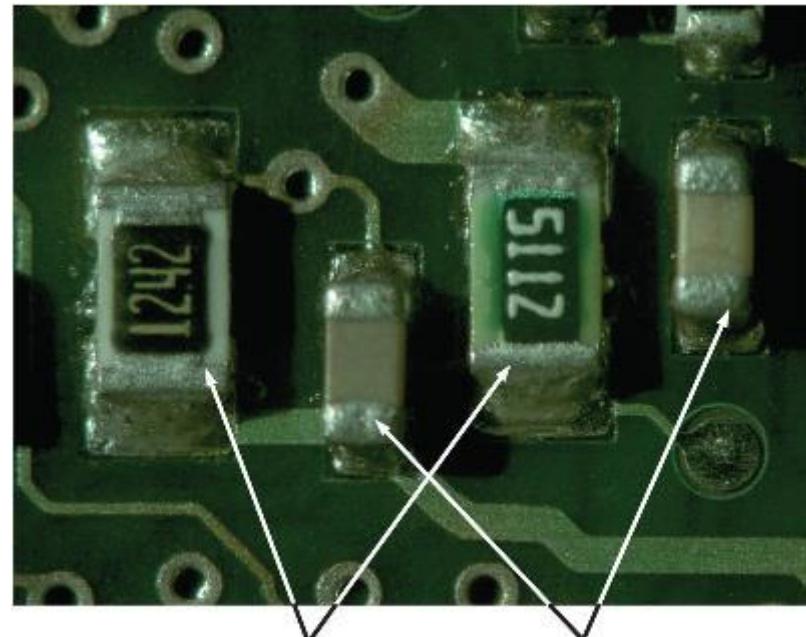
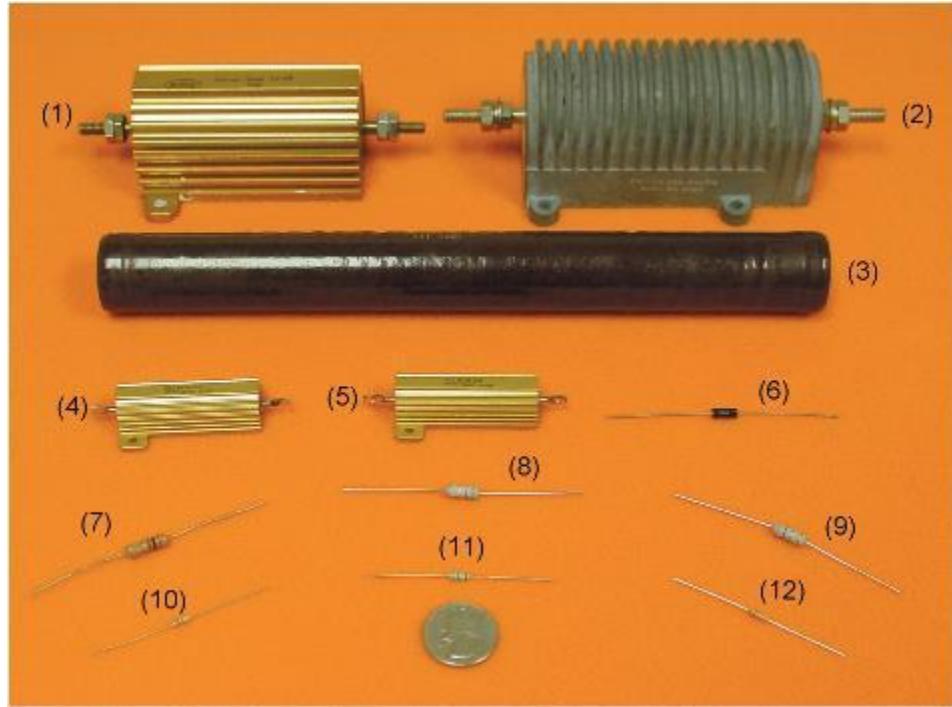
(a) Direct current



(b) Alternating current

*Please refer to “*The War of Currents*” reading provided under Supplementary Material on [Brightspace](#).

Resistors



Chip Resistors

Chip Capacitors

Resistors: Notation and Colour Coding

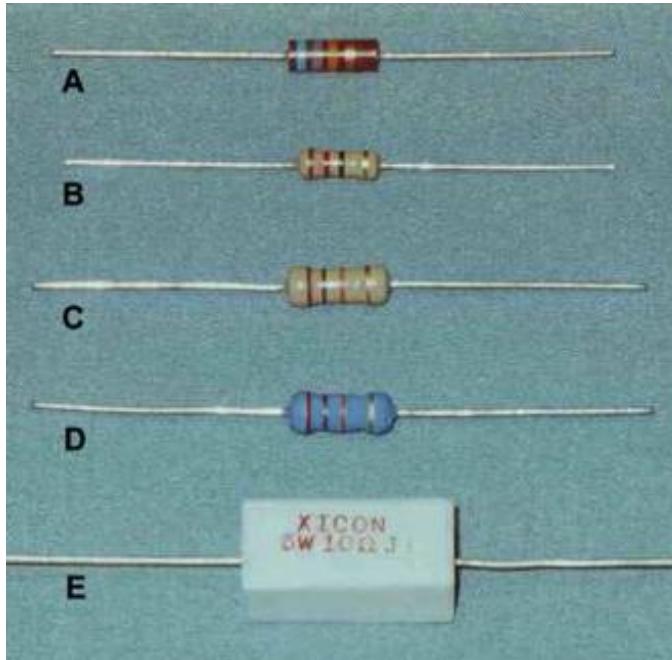
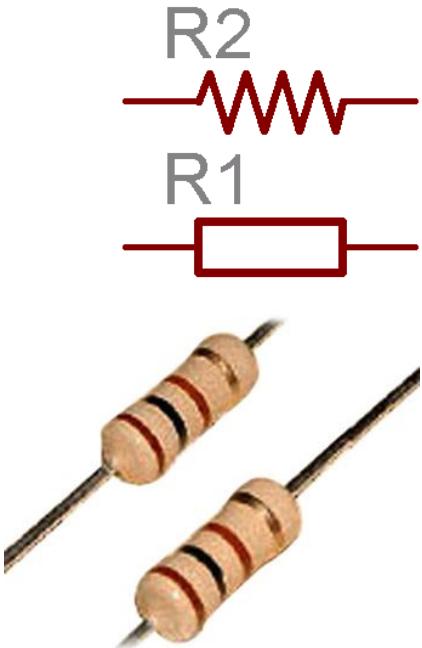


Diagram illustrating resistor color coding. It shows a 4-band resistor with a value of $560\text{k}\Omega \pm 5\%$ and a 5-band resistor with a value of $237\Omega \pm 1\%$. The 4-band code table is as follows:

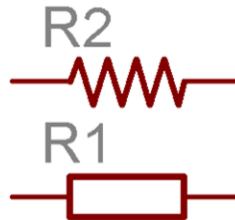
COLOR	1 ST BAND	2 ND BAND	3 RD BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	$\pm 1\%$ (F)
Red	2	2	2	100Ω	$\pm 2\%$ (G)
Orange	3	3	3	$1\text{K}\Omega$	
Yellow	4	4	4	$10\text{K}\Omega$	
Green	5	5	5	$100\text{K}\Omega$	$\pm 0.5\%$ (D)
Blue	6	6	6	$1\text{M}\Omega$	$\pm 0.25\%$ (C)
Violet	7	7	7	$10\text{M}\Omega$	$\pm 0.10\%$ (B)
Grey	8	8	8	$100\text{M}\Omega$	$\pm 0.05\%$
White	9	9	9	$1\text{G}\Omega$	
Gold				0.1Ω	$\pm 5\%$ (J)
Silver				0.01Ω	$\pm 10\%$ (K)

The 5-band code table is as follows:

CODE	1 ST BAND	2 ND BAND	3 RD BAND	4 TH BAND	5 TH BAND	RESISTANCE	TOLERANCE
0.1%, 0.25%, 0.5%, 1%	0	0	0	0	0	237Ω	$\pm 1\%$

Resistors and Resistance

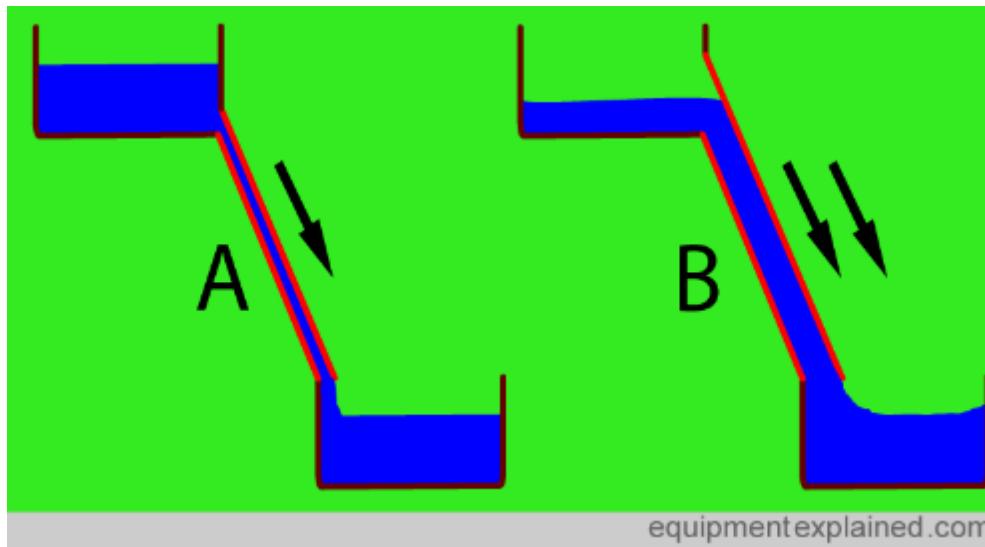
In (linear) circuits **resistors** are components typically represented by these symbols:



They model the material physical property of **resistance**, i.e., the physical property or ability to resist the flow, the passage of electric current through them.

Resistance: Water Analogy

If the current is a flow of water, higher resistance is represented by a narrower pipe...



Resistance

The resistance is measured in **Ohms** and is given by

$$R = \frac{\rho L}{A}$$

ρ : resistivity

L : length of the sample

A : cross-sectional area of the sample

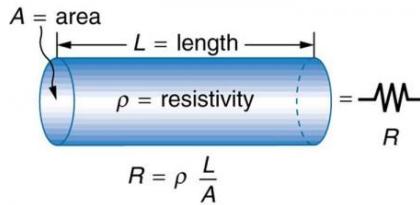


TABLE 12.1

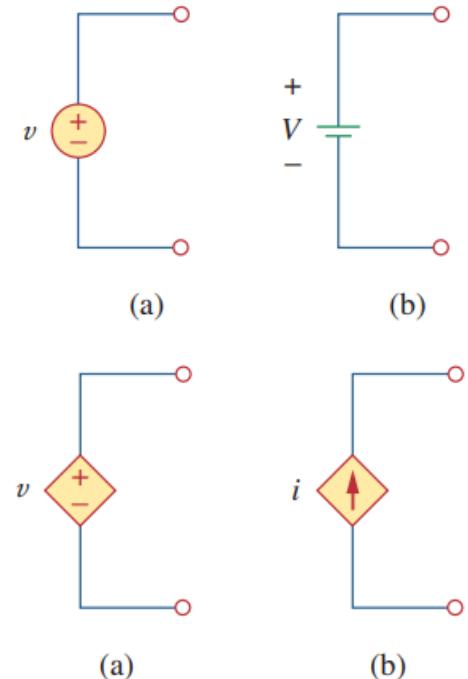
Electrical Resistance for 1 m long Wire
Made of Various Metals Having an Area
of 1 m² at 20° C

Metal	Resistance ($\times 10^{-8}$ ohms, Ω)
Aluminum	2.82
Brass	17.2
Copper	1.68
Gold	2.44
Iron	10
Lead	22
Nickel	6.99
Platinum	10.6
Silver	1.59
Tin	10.9
Tungsten	5.6
Zinc	5.9

Saeed Moaveni, *Engineering Fundamentals: An Introduction to Engineering*, Cengage Learning, 4th Ed., 2010.

Independent and Dependent Sources

- **Independent source** → active element providing specified voltage or current that is independent of other circuit elements
- **Dependent (or controlled) source** → active element controlled by another voltage or current:
 1. A voltage-controlled voltage source (VCVS).
 2. A current-controlled voltage source (CCVS).
 3. A voltage-controlled current source (VCCS).
 4. A current-controlled current source (CCCS).



Week 1.2

Overview of today

- Ohm's law
- Resistance and conductance
- Short circuit, open circuit
- Kirchhoff's laws
- Series and parallel connections
- Voltage division, current division (duality)

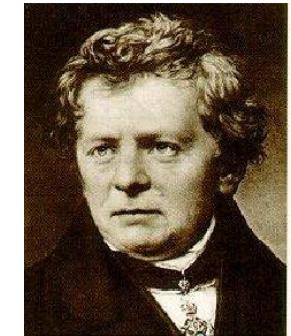
Ohm's law

- In general, we want to link current through & voltage across a component by a function F so that:

$$v(t) = F[i(t)]$$

- For a linear resistance holds Ohm's law:

$$v = i \cdot R$$



Georg Simon Ohm:

- The measure unit for resistance is **ohm**, Ω . 1787 – 1854

Conductance

- We can also express the current as a function of the voltage by inverting the previous equation. The inverse of the resistance R is the so-called *conductance* G .

$$i = \frac{1}{R} \cdot v = G \cdot v$$

- The measure unit is **siemens**, S.

Resistance and Power

- Resistors are **passive** network elements that can only dissipate energy.
- By making use of: $p = v \cdot i$

$$v = i \cdot R$$

- It follows that:

$$v = p / i \quad i = v / R \quad i = \sqrt{p / R}$$

$$R = v / i$$

$$p = v^2 / R$$

$$p = R \cdot i^2$$



Power: your turn!

$$v = p / i$$

$$R = v / i$$

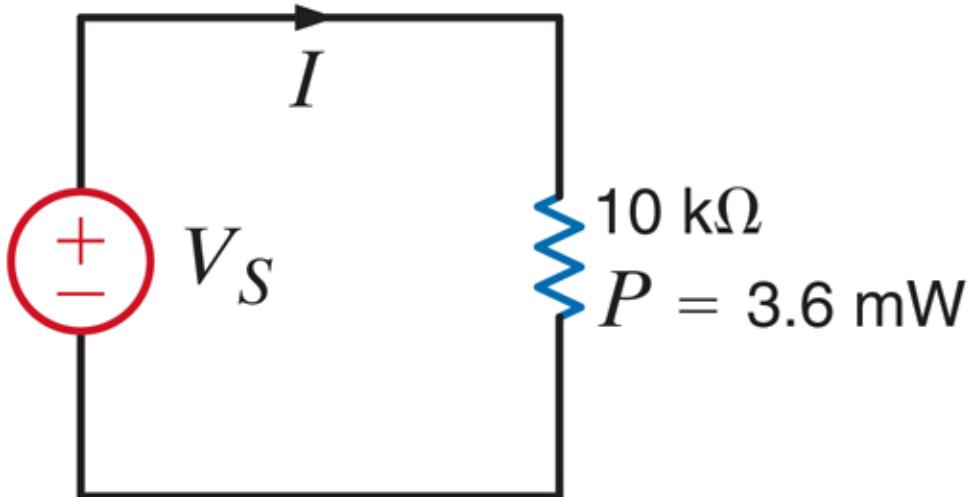
$$i = v / R$$

$$p = v^2 / R$$

$$i = \sqrt{p / R}$$

$$p = R \cdot i^2$$

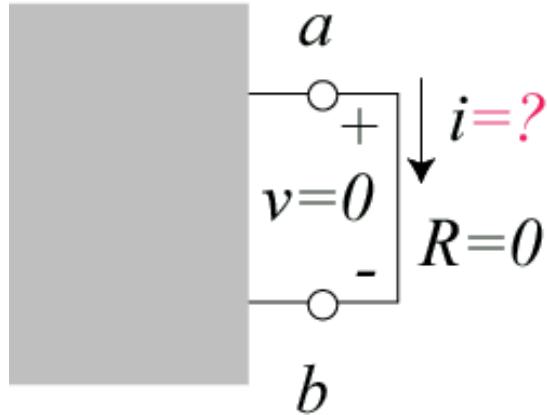
Calculate the voltage V_S and the current I





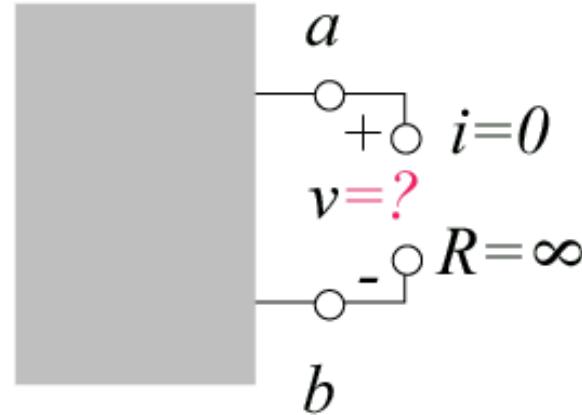
Short and Open Circuit

Short Circuit



What is the current i through the short circuit?

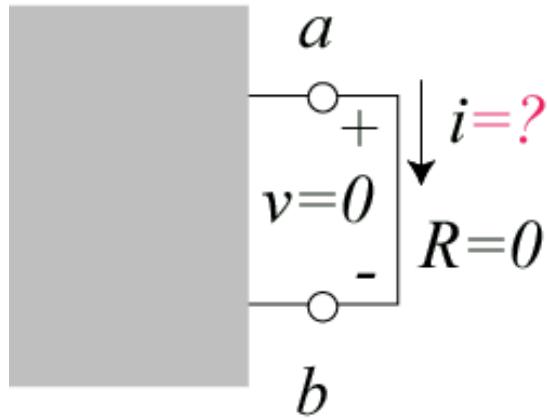
Open Circuit



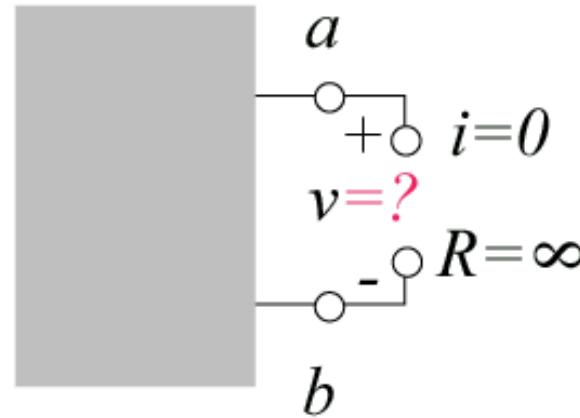
What is the voltage v_{ab} across the open circuit?

Short and Open Circuit

Short Circuit



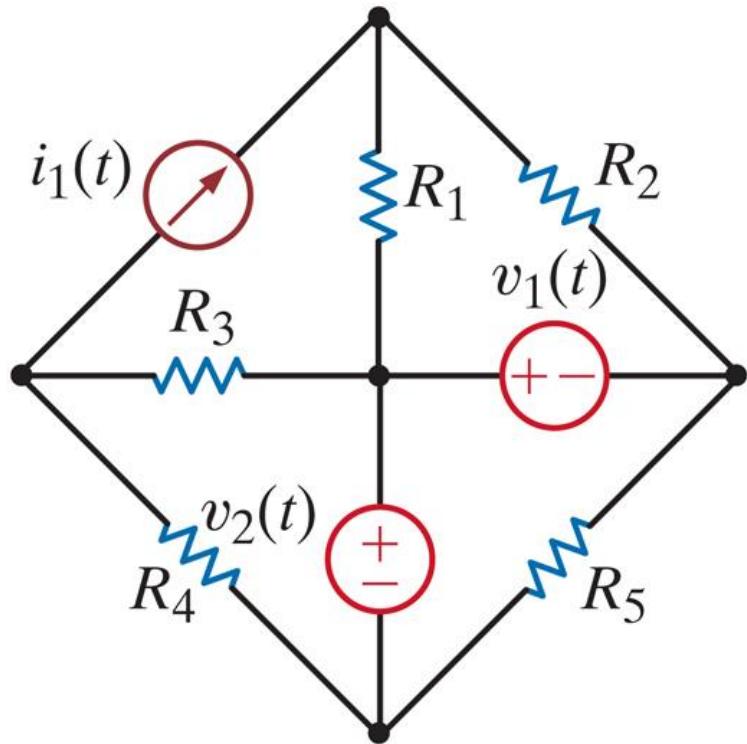
Open Circuit



Neither the current through a short circuit nor the voltage across an open circuit can be calculated using Ohm's Law.

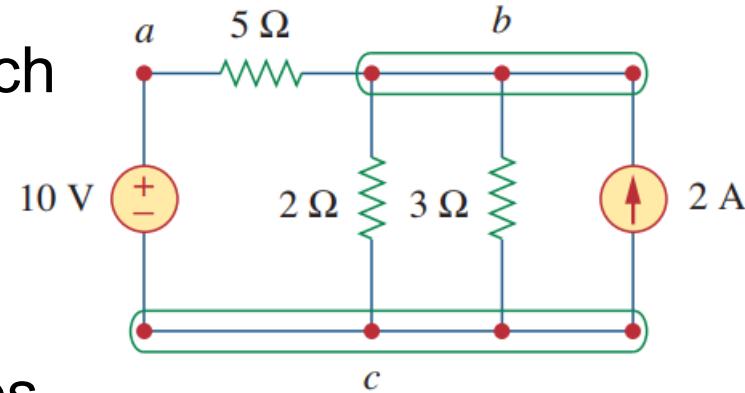
-> In this case they depend on the other (grey) area of the circuit.

Multiple Interconnections: From Components to Circuits

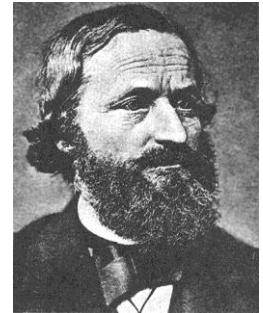


First, important vocabulary

- **Branch** → Single element such as a voltage source or a resistor.
- **Node** → Point of connection between two or more branches.
- **Loop** → Any closed path in a circuit.
- **Mesh** → A loop that does not enclose other loops.



Then, the fundamental rules: Kirchhoff's laws



Kirchhoff's Current Law (KCL):

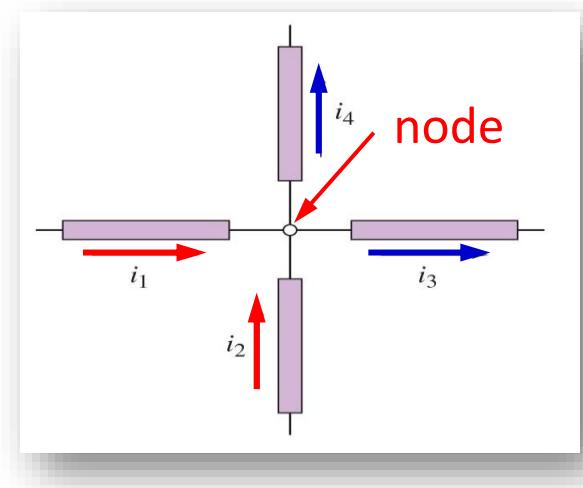
- “The sum of all currents in a node is zero”

$$\sum_{j=1}^n i_j(t) = 0$$

- Or... “The sum of all currents entering a node is equal to the sum of all currents exiting a node”.

$$i_1 + i_2 = i_3 + i_4$$

- KCL comes from the indestructibility of electric current: you cannot create/destroy electrons.



Then, the fundamental rules: Kirchhoff's laws

Kirchhoff's Voltage Law (KVL):

- The sum of all voltages in a mesh/loop is zero:

$$\sum_{j=1}^n v_j(t) = 0$$

- As we follow any loop in a circuit in a specific direction, some voltages across components will be negative, some positive, but when we finish the loop, the sum will be zero.
- ✓ These laws are based on the general conservation laws: conservation of charge (KCL) and conservation of energy (KVL).

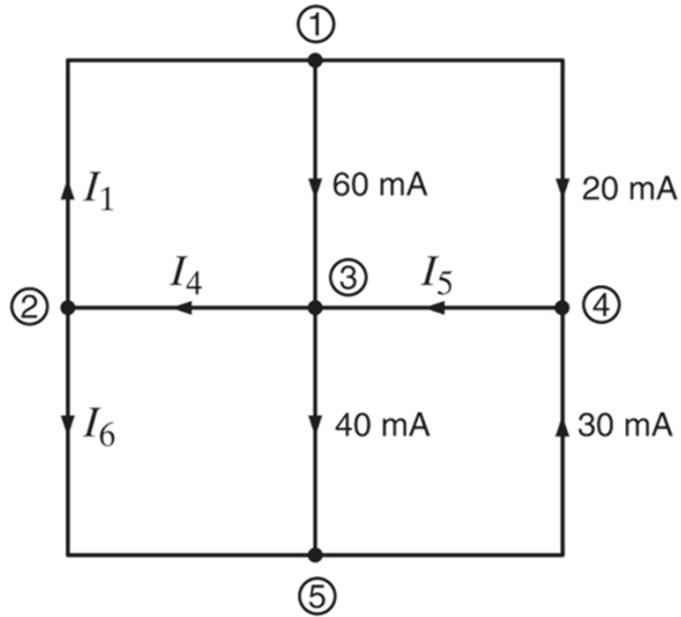




Example: KCL

(This example was given at the SKC dag if you attended it...)

Calculate the value of the current I_1 and I_6

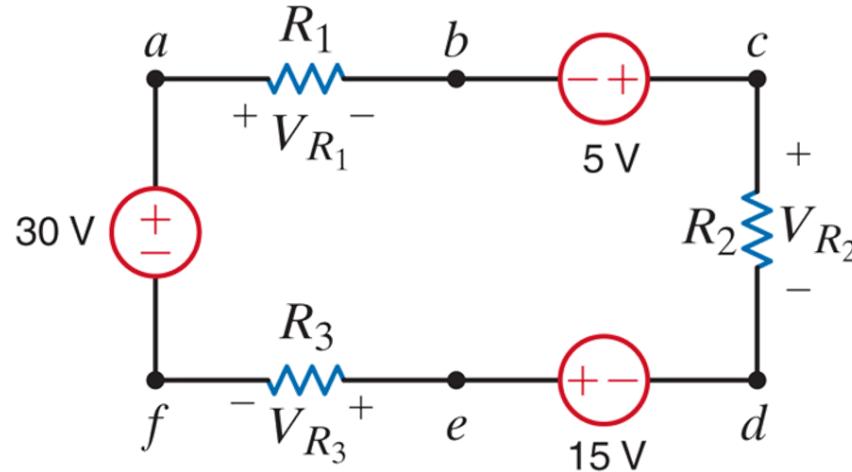




Example: KVL

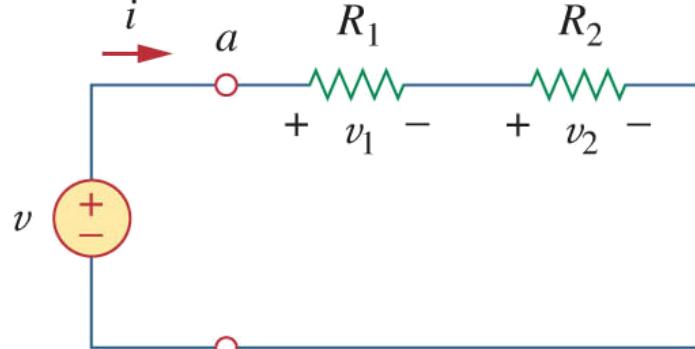
(This example was given at the SKC dag if you attended it...)

Calculate the value of the voltage V_{R_3} if $V_{R_1} = 18V$ and $V_{R_2} = 12V$



Series and Parallel Connections

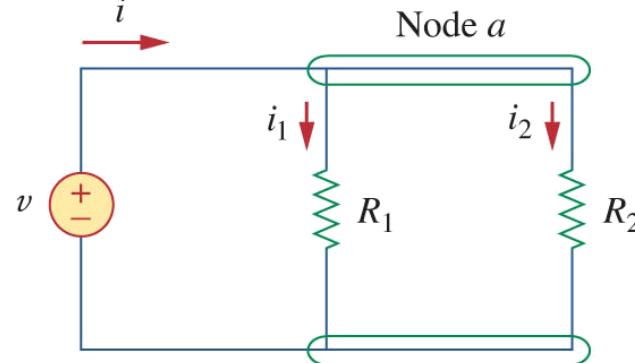
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Series circuits: b

- The same current through each element
- Voltage division -> the voltage is divided across the different components

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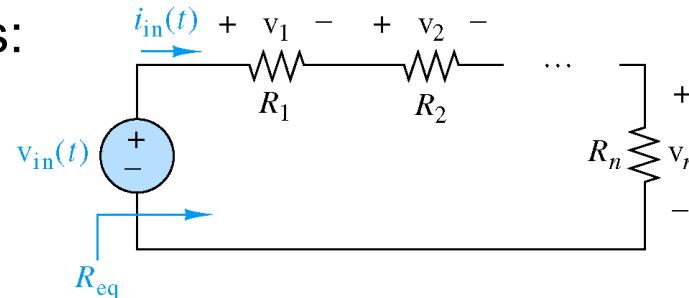
Parallel circuits:

- The same voltage over each element
- Current division -> the current is divided through the different components

Resistors in Series and Parallel

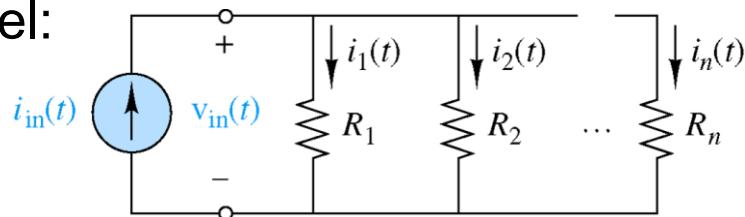
- **Equivalent resistance** in series:

$$R_{eq} = R_1 + R_2 + \dots + R_n$$



- **Equivalent resistance** in parallel:

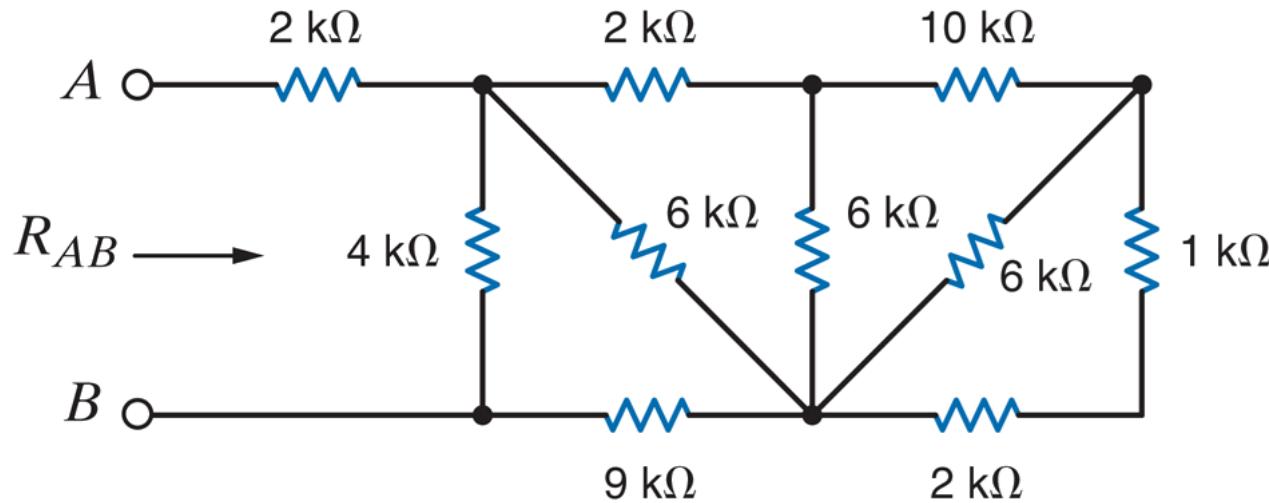
$$\begin{aligned} R_{eq} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} \\ &= \frac{1}{G_1 + G_2 + \dots + G_n} = \frac{1}{G_{eq}} \end{aligned}$$



Note that, in the case of parallel circuits, using conductance is more convenient!

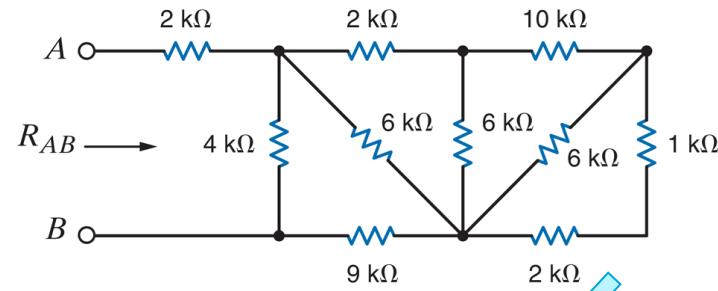
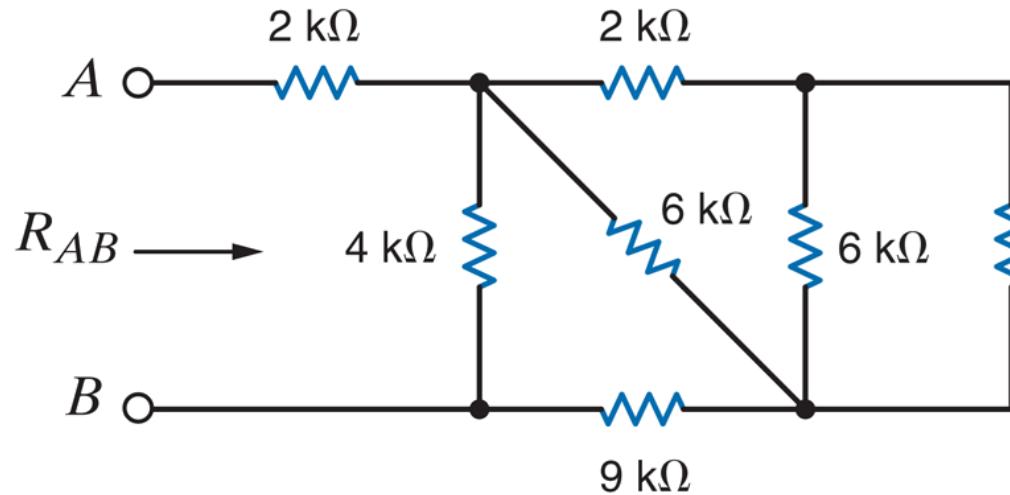


(Exam)ple: Resistors in Series and Parallel

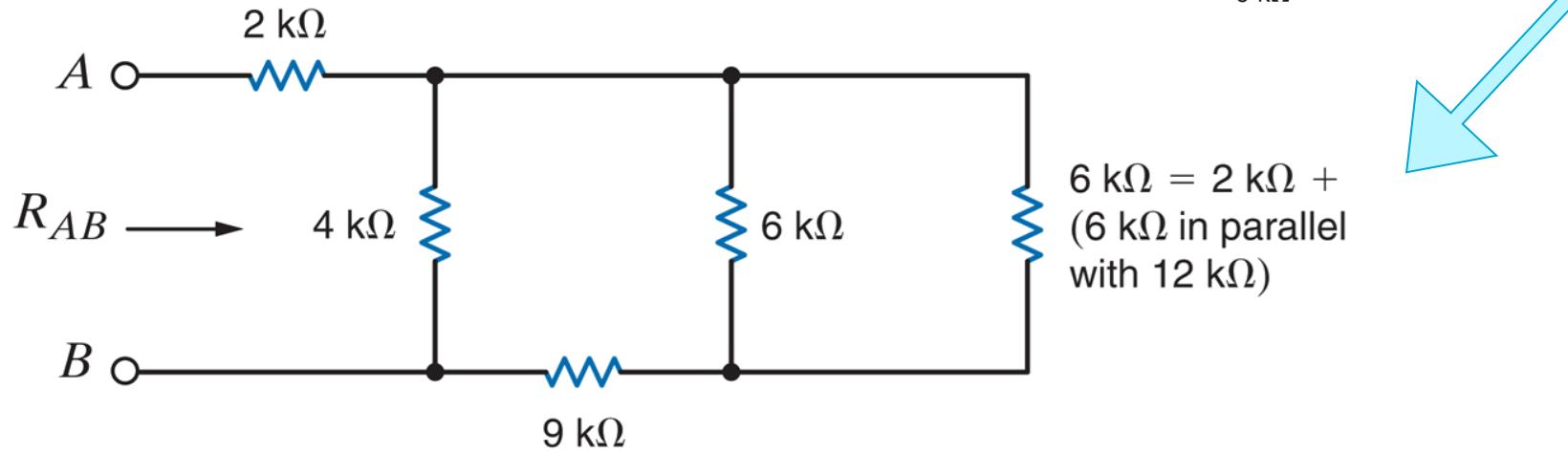
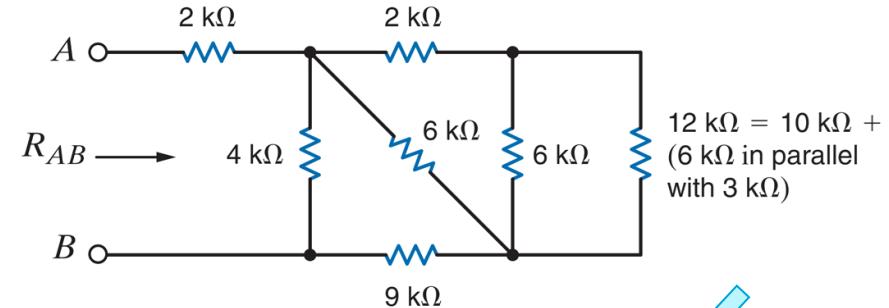


Calculate the equivalent resistance R_{AB} ?

(Exam)ple: Resistors in Series and Parallel

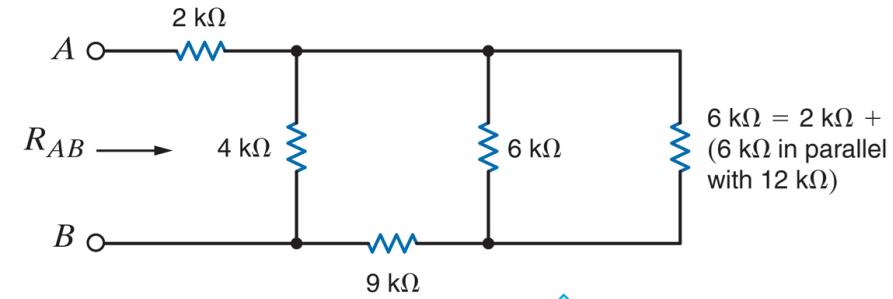
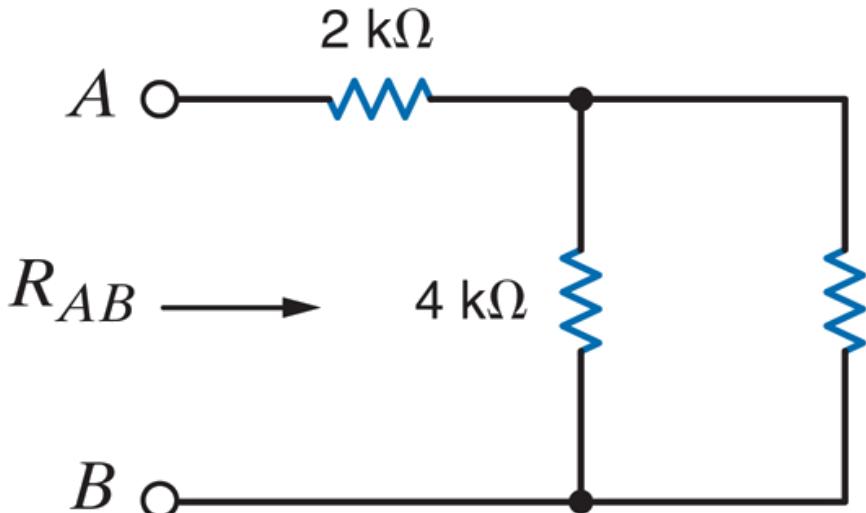


(Exam)ple: Resistors in Series and Parallel



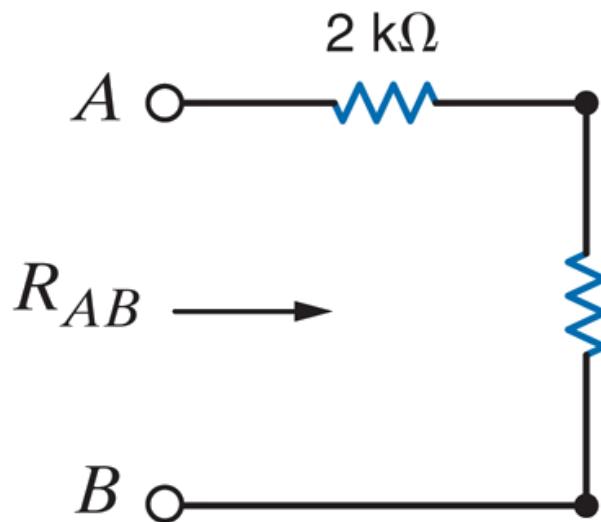
$$6 \text{ k}\Omega = 2 \text{ k}\Omega + (6 \text{ k}\Omega \text{ in parallel with } 12 \text{ k}\Omega)$$

(Exam)ple: Resistors in Series and Parallel



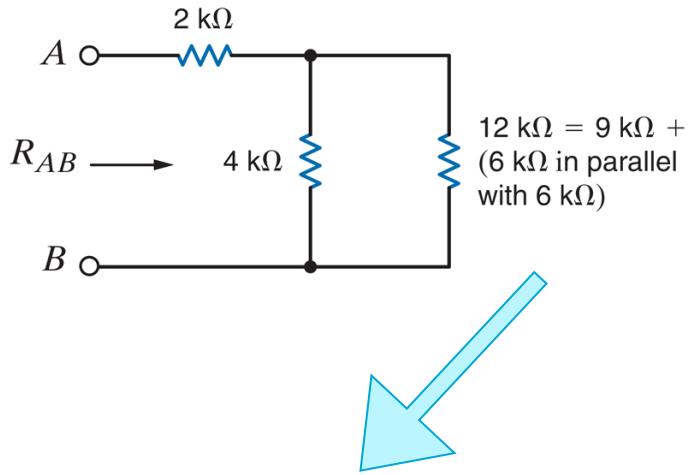
$$12\text{ k}\Omega = 9\text{ k}\Omega + (6\text{ k}\Omega \text{ in parallel with } 6\text{ k}\Omega)$$

(Exam)ple: Resistors in Series and Parallel

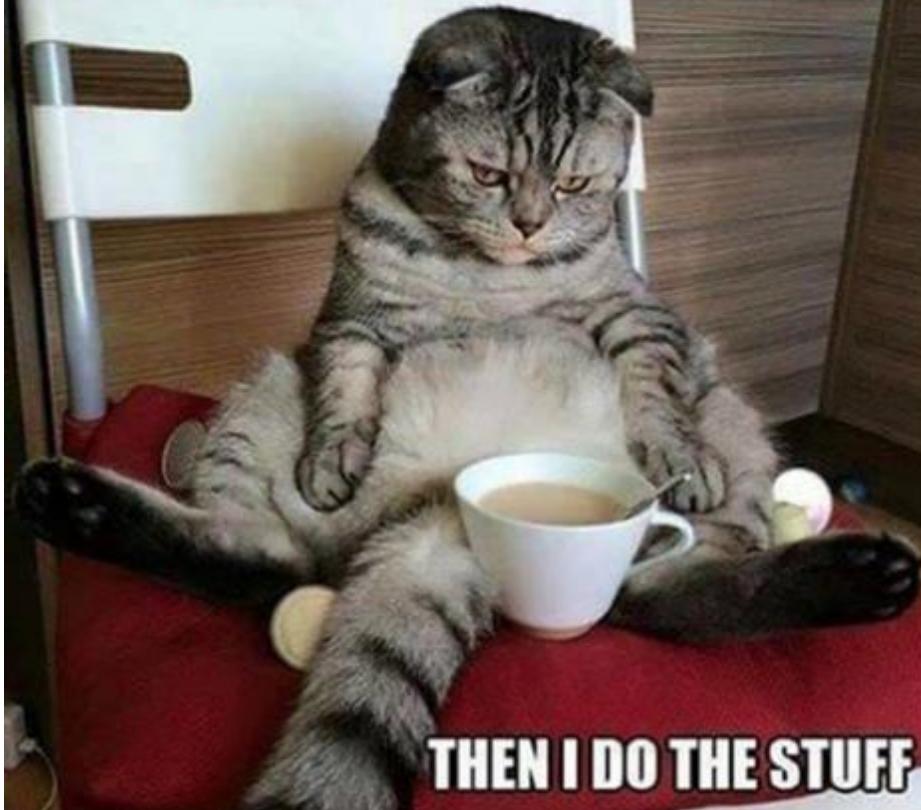


$3\text{ k}\Omega = (4\text{ k}\Omega \text{ in parallel with } 12\text{ k}\Omega)$

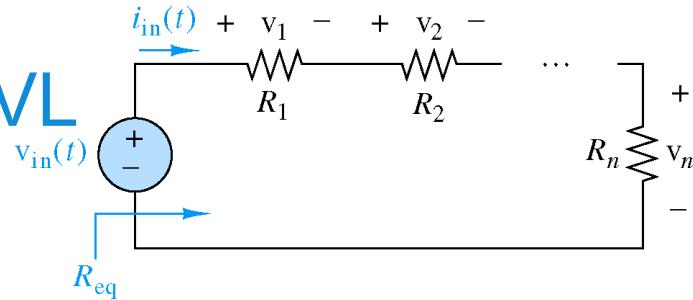
So, eventually $R_{AB} = \dots$



FIRST I DRINK THE COFFEE



Focus: Resistors in Series & KVL



KVL: $V_{in}(t) = V_1 + V_2 + \dots + V_n$

Ohm's law: $V_{in}(t) = R_1 i_{in}(t) + R_2 i_{in}(t) + \dots + R_n i_{in}(t) = R_{eq} i_{in}(t)$, where $R_{eq} = \sum_{j=1}^n R_j$

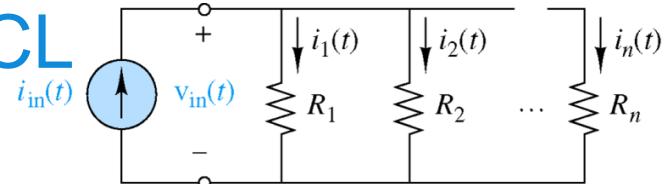
Inverting the formula, we get $i_{in}(t) = V_{in}(t)/R_{eq}$

If now we want to find the voltage $V_j(t)$ on one resistor R_j ? We use Ohm's law and the fact that the current is the same for all resistors (series connection). Hence:

$$V_j(t) = R_j i_{in}(t) = V_{in}(t) \frac{R_j}{R_{eq}} = V_{in}(t) \frac{R_j}{\sum_{j=1}^n R_j}$$

This is the so-called **voltage divider equation**, as it divides the input voltage across the different resistors connected in series.

Focus: Resistors in Parallel & KCL



KCL: $I_{in}(t) = I_1 + I_2 + \dots + I_n$

Ohm's law: $I_{in}(t) = V_{in}(t)/R_1 + V_{in}(t)/R_2 + \dots + V_{in}(t)/R_n = V_{in}(t)(1/R_{eq})$, where $1/R_{eq} = \sum_{j=1}^n 1/R_j$

Inverting the formula, we get $V_{in}(t) = i_{in}(t)R_{eq}$

If now we want to find the current $I_j(t)$ on one resistor R_j ? We use Ohm's law and the fact that the voltage is the same for all resistors (parallel connection). Hence:

$$I_j(t) = \frac{V_{in}(t)}{R_j} = i_{in}(t) \frac{R_{eq}}{R_j} = i_{in}(t) \frac{(\sum_{j=1}^n 1/R_j)^{-1}}{R_j} = i_{in}(t) \frac{1}{R_j} \frac{1}{\sum_{j=1}^n 1/R_j} = i_{in}(t) \frac{\frac{1}{R_j}}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

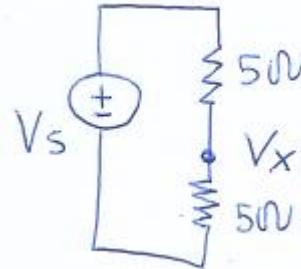
This is the so-called **current divider equation**, as it divides the input current through the different resistors connected in parallel.

Maths is trickier. But if you write the formulae with conductances instead of resistances is easier.

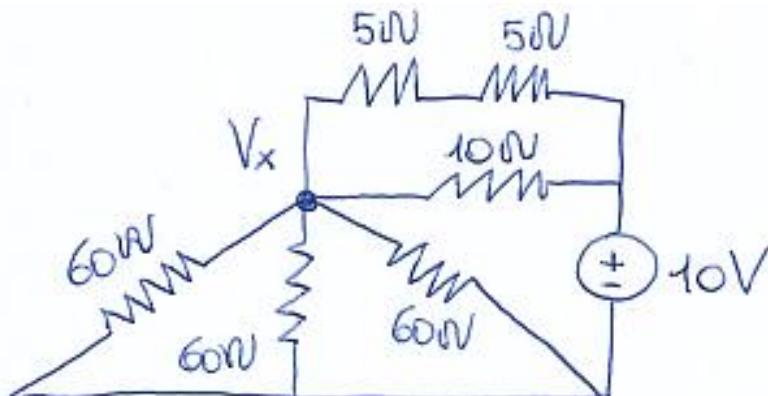


Your turn now

Consider the circuit on the right-hand side and calculate V_x .



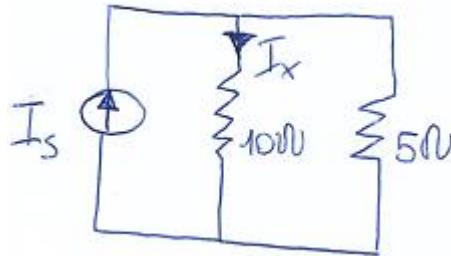
Now consider the circuit below and calculate V_x . Do not freeze, look at the circuit for simplifications.



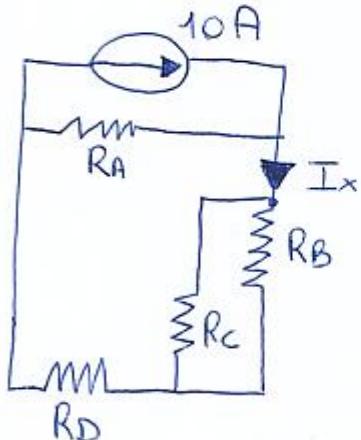


Your turn now

Calculate the current I_x in the circuit on the right-hand side.



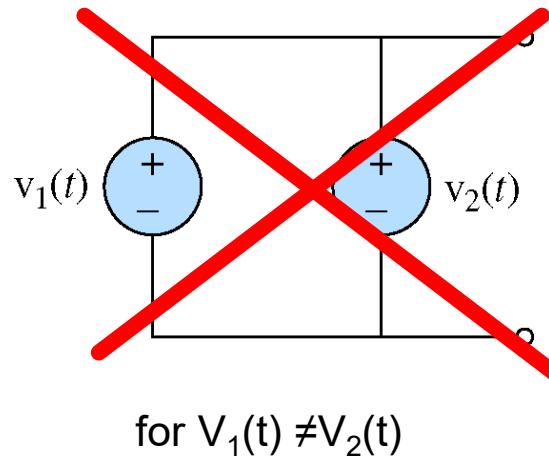
Now consider the circuit below and calculate I_x .



$$R_B = R_C = 6\Omega$$
$$R_D = 2\Omega$$
$$R_A = 5\Omega$$

Focus: Voltage Sources in Parallel? No

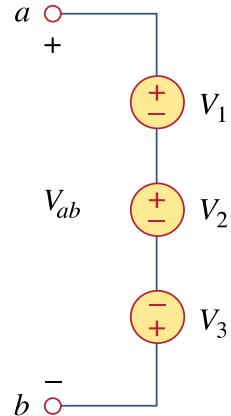
Two ideal voltage sources (with different generator voltages) in parallel are conflicting with Kirchhoff's voltage law!



Focus: Voltage sources in series? Yes

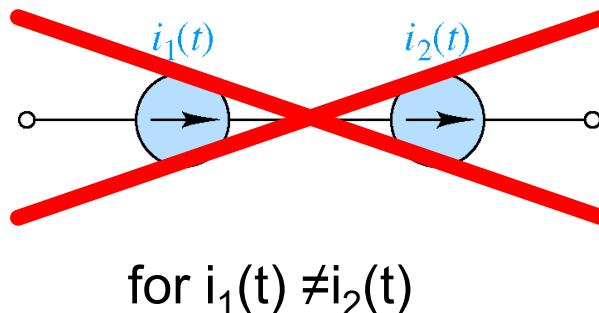
The voltage of components connected in series add up (with sign!).

$$V_{ab} = V_1 + V_2 - V_3$$



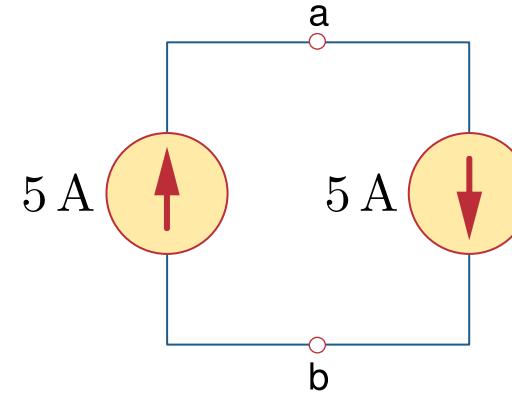
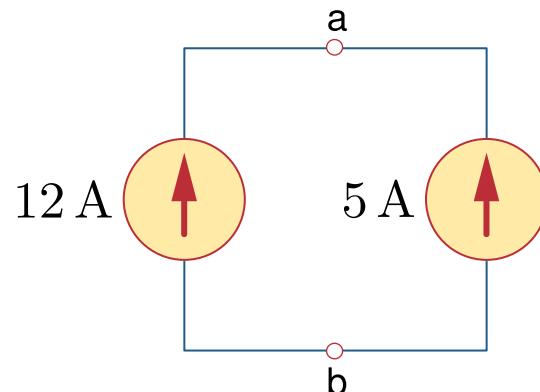
Focus: Current Sources in Series? No

Two ideal current sources (with different generator currents) in series are conflicting with Kirchhoff's current law!



Focus: Current Sources in Parallel? It depends...

Ideal current sources (with different generator currents) in parallel might conflict with Kirchhoff's current law. You have to be careful with the interconnections. More in the instructions...



Summary

- Ohm's law
- Resistance and conductance
- Short circuit, open circuit
- Kirchhoff's laws
- Series and parallel connections
- Voltage division, current division (duality) -> more in the Tuesday/Friday seminars with instructions.

Next steps (see in case the syllabus)

- **SGH** (Self-Graded Homework assignments): posted today; submission due on Wednesday.
- **Seminar**: in groups on Tuesday & altogether on Friday.
- **Next week**: Circuit Analysis
 - Nodal analysis
 - Mesh analysis

Thank you!