EE1C1 “Linear Circuits A”

Week 1.3

Francesco Fioranelli/ Ioan E. Lager

My e-mail address: F.Fioranelli@tudelft.nl / room 20.280 (20th floor of EWI building)
My group website: radar.tudelft.nl

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Faculty of Electrical Engineering, Mathematics and Computer Science

Today

- Recap: Ohm's law, KVL/KCL
- Week 1.3:
  - Methods for circuit analysis
    - Visual Inspection
    - Nodal analysis
    - Mesh analysis
  - Suitability of each method
- Summary and Next Week

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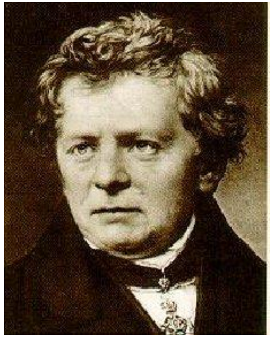
Recap

Ohm’s Law, KVL, KCL

- Ohm’s law holds for a linear resistance:

The measure unit for resistance is ohm, Ω

$$v = i \cdot R$$



Kirchhoff’s Current Law (**KCL**):

- The sum of all currents in a node is zero.

$$\sum_{j=1}^{n} i_j(t) = 0$$

Kirchhoff’s Voltage Law (**KVL**):

- The sum of all voltages in a mesh is zero.

$$\sum_{j=1}^{n} v_j(t) = 0$$

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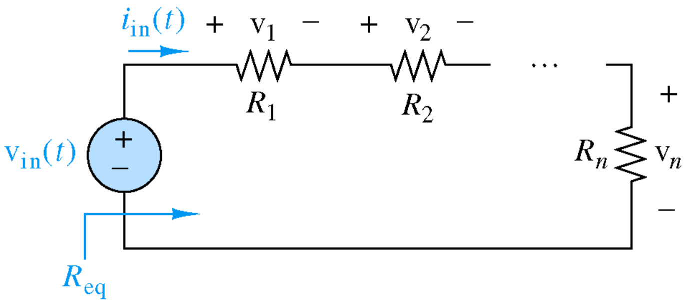
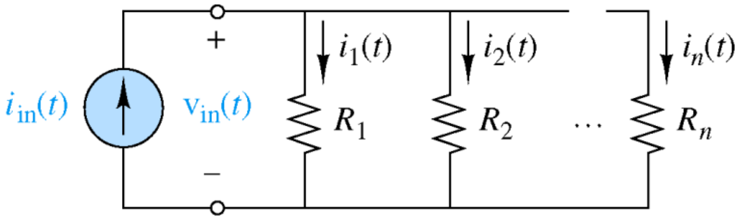
Series and Parallel Connections

- Voltage (left) and current (right) dividers, respectively:



$$R_{eq} = R_1 + R_2 + ... + R_n$$

$$v_j(t) = \frac{R_j}{R_1 + R_2 + ... + R_n} v_{in}(t) = \frac{R_j}{R_{eq}} v_{in}(t)$$



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}}$$

$$i_j(t) = \frac{\frac{1}{R_j}}{\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}} i_{in}(t)$$

Week 1.3

Please mind that…

Linear Circuits in Week 1

Linear Circuits in Week 2

Linear Circuits in Week 3 and onward

TUDelft *So, get your s…trategy together to approach this course!*

Analysing an electric circuit…

This means in first instance being able to **calculate the voltages and currents at each branch**.

-You may be asked to calculate a numerical value (with unit)

-You may be asked to write an equation that links the different quantities (without numbers, but with variables)

How to do it?

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Analysing an electric circuit…

First, we need to be **aware of the tools** we have in our toolbox, so far:

-Ohm’s law

-KCL

-KVL

-calculation of equivalent resistance in series/parallel

-voltage/current dividers

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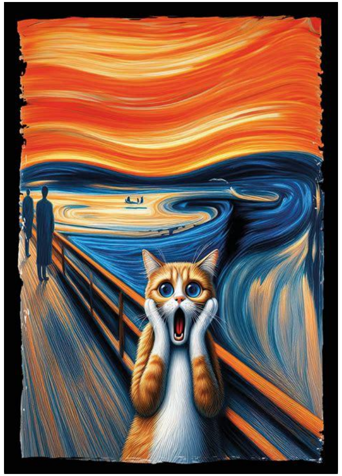
Method of Visual Inspection

Literally, **look** at the circuit and…

a) see if there are simplifications to make (e.g., equivalent resistances)

b) start writing the relevant equations (Ohm, Kirchhoff, V/I dividers)

Method of Visual Inspection

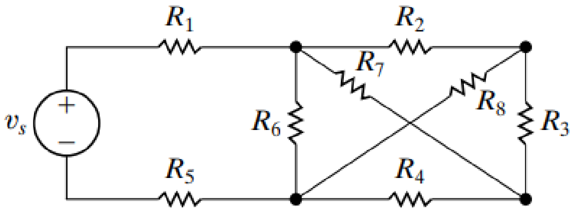


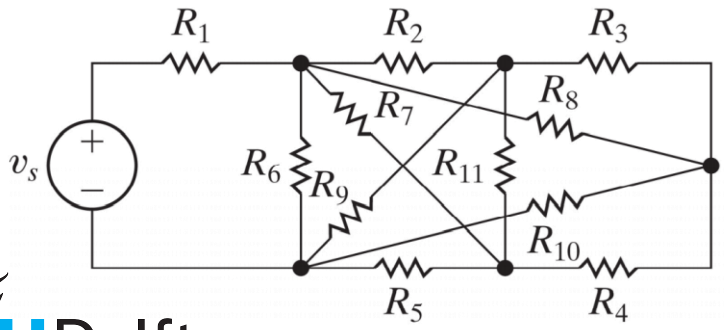
- Requires experience and insight
- Is difficult to implement algorithmically
- Can fail if the circuit increases in complexity

We prefer a **generally applicable method** working according to an algorithm! Let's look at them…

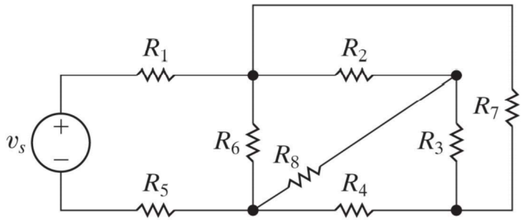
Planar Networks

First though a definition. **Planar** circuits → on a plane with no crossing branches.





Non-planar



Planar

Nodal Analysis

Nodal Analysis - Steps

Step 1: Bookkeeping:

- Determine the number of nodes (N)
- Choose the reference (the ground)
- Draw the current directions in each circuit branch

LINEAR CIRCUITS…

Step 2: Assemble the needed equations:

- Assemble the Kirchhoff current equations for all nodes except the ground (N-1 equations)
- Express the currents in terms of voltages (KCL)

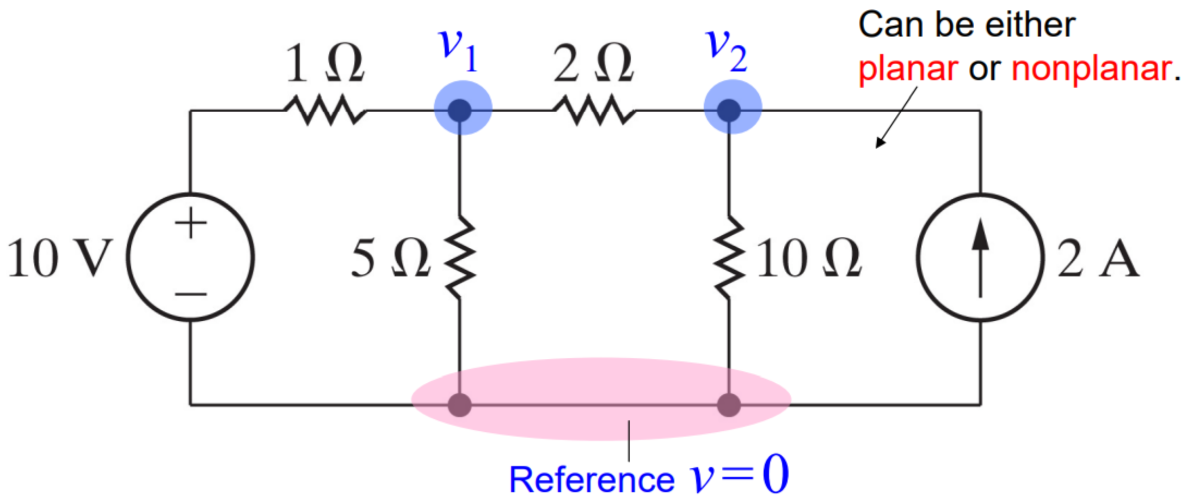
Step 3: Solve the equations

- Solve the system of linear equations

MATHS…

Note: This method works with planar and non-planar circuits.

Nodal Analysis: example 1



- Select one of the essential nodes, i.e. a node where three or more circuit elements join. → revisit the definition of reference and non-reference nodes.

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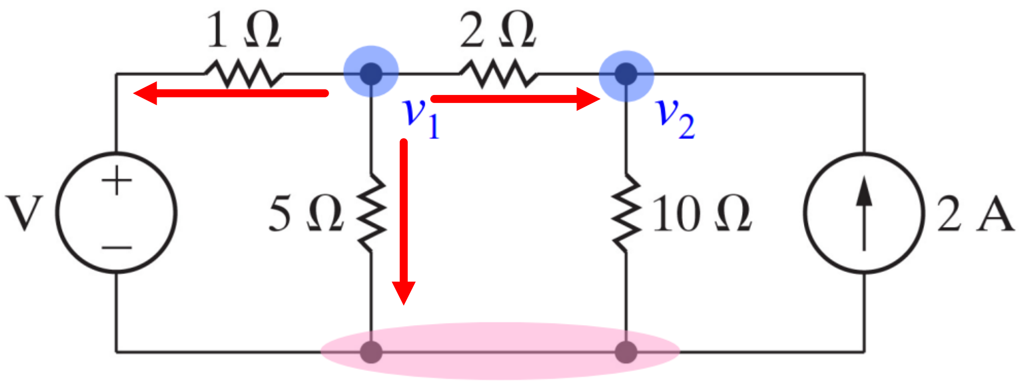
Nodal Analysis: example 1

- Write the system of KCL equations

Node 1: $\left\{\frac{v_1-10}{1\,\Omega}+\frac{v_1}{5\,\Omega}+\frac{v_1-v_2}{2\,\Omega}=0,\right.$

Node 2: $\frac{v_2-v_1}{2\,\Omega}+\frac{v_2}{10\,\Omega}=2.$

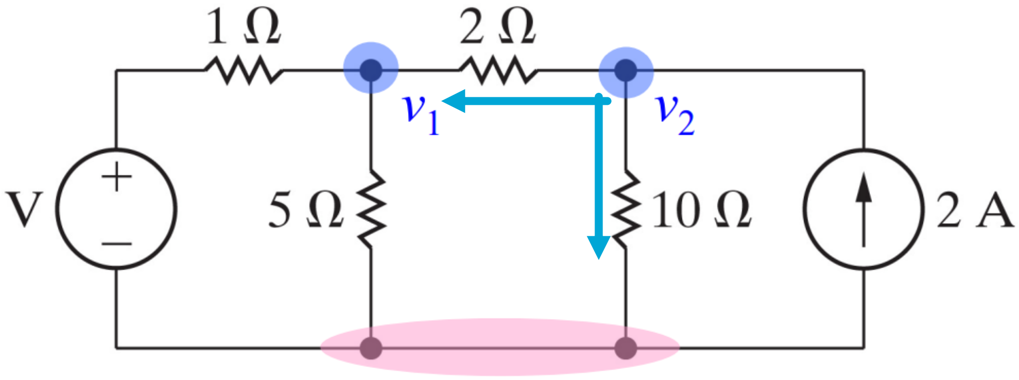


Nodal Analysis: example 1

- Write the system of KCL equations

Node 1: $\begin{cases} \dfrac{v_1 - 10}{1\,\Omega} + \dfrac{v_1}{5\,\Omega} + \dfrac{v_1 - v_2}{2\,\Omega} = 0, \\ \dfrac{v_2 - v_1}{2\,\Omega} + \dfrac{v_2}{10\,\Omega} = 2. \end{cases}$

Node 2:

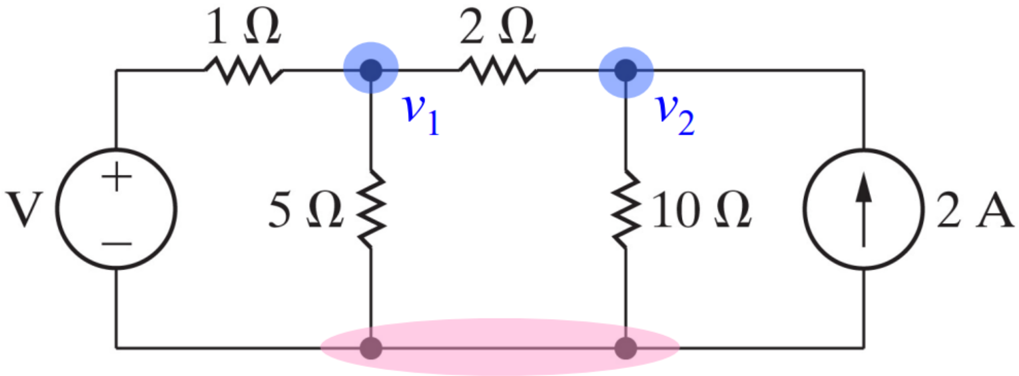


Nodal Analysis: example 1

- Solve the system, with the technique you prefer

Node 1: $\left\{ \frac{v_1 - 10}{1\,\Omega} + \frac{v_1}{5\,\Omega} + \frac{v_1 - v_2}{2\,\Omega} = 0, \right.$

Node 2: $\frac{v_2 - v_1}{2\,\Omega} + \frac{v_2}{10\,\Omega} = 2.$



$$\begin{cases} 10v_1 - 100 + 2v_1 + 5v_1 - 5v_2 = 0 \\ 5v_2 - 5v_1 + v_2 = 20 \end{cases} \qquad \begin{cases} \boldsymbol{v_1 = 9{,}09\ V} \\ \boldsymbol{v_2 = 10{,}9\ V} \end{cases}$$

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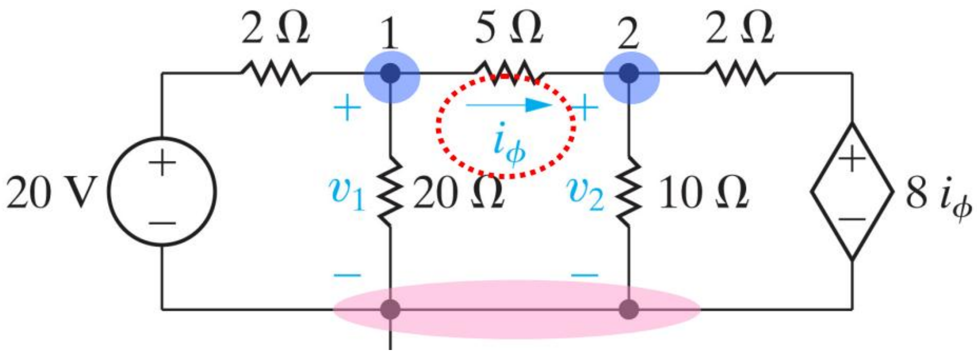
Nodal analysis: circuits with dependent sources



$$\begin{cases} \text{Node 1: } \dfrac{v_1 - 20}{2\,\Omega} + \dfrac{v_1}{20\,\Omega} + \dfrac{v_1 - v_2}{5\,\Omega} = 0, \\ \text{Node 2: } \dfrac{v_2 - v_1}{5\,\Omega} + \dfrac{v_2}{10\,\Omega} + \dfrac{v_2 - 8i_\phi}{2\,\Omega} = 0, \\ \text{Constraint: } i_\phi = \dfrac{v_1 - v_2}{5\,\Omega}. \end{cases}$$

$$\Rightarrow \begin{bmatrix} 0.75 & -0.2 \\ -1 & 1.6 \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix} \text{V}, \; i_\phi = 1.2 \text{ A}.$$

An extra constraint means a system with 3 equations to solve.

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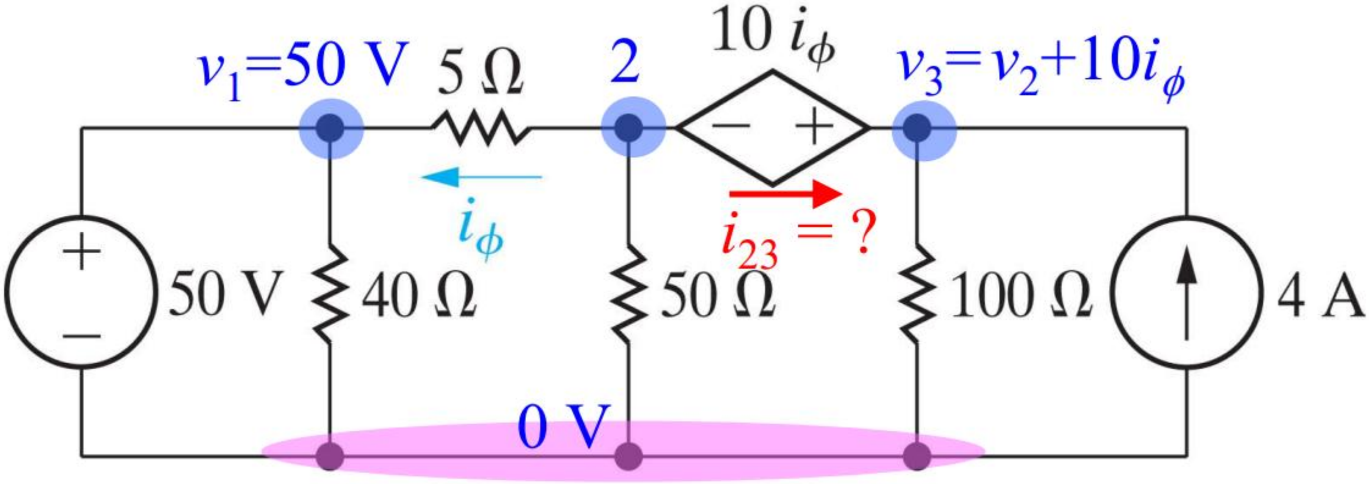
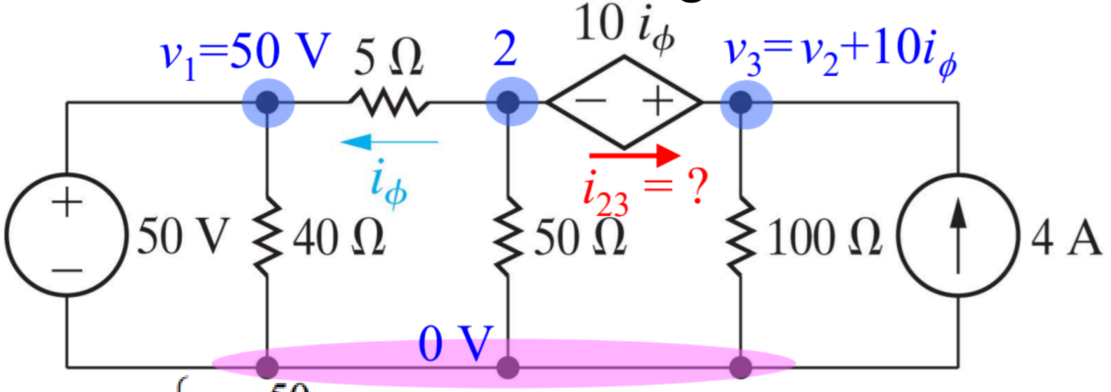
Nodal analysis: voltage source between 2 nodes

- When an independent or a dependent voltage source is the only element between two nodes → the branch current is undetermined!
- We cannot apply KCL to those nodes.
- What now?!



Nodal analysis: voltage source between 2 nodes

Apply the elimination method to remove one variable; then use the constraint that comes from the voltage source.



Node 2: $\frac{v_2 - 50}{5\,\Omega} + \frac{v_2}{50\,\Omega} + i_{23} = 0,$

Node 3: $\frac{v_3}{100\,\Omega} = i_{23} + 4.$

$$\Rightarrow \frac{v_2 - 50}{5\,\Omega} + \frac{v_2}{50\,\Omega} + \frac{v_3}{100\,\Omega} = 4$$

Source constraint: $v_3 = v_2 + 10(v_2 - v_1)/(5\,\Omega)$

Still missing an equation? Yes, the KCL at the node 1…

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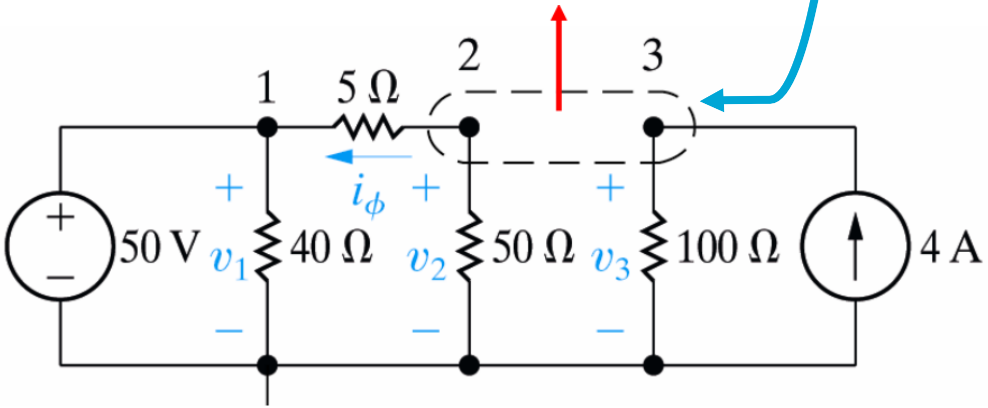
Nodal analysis: voltage source between 2 nodes

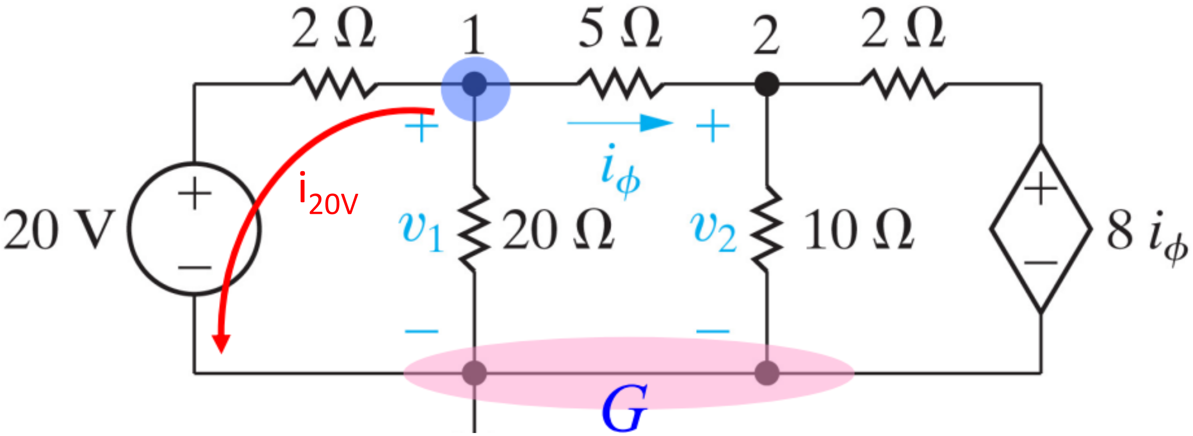
- Alternative approach: apply KCL to a **supernode** formed by the combination of two essential nodes; one can get the same equation as in the previous slide without the intermediate elimination step.

$$\frac{v_2 - 50}{5\,\Omega} + \frac{v_2}{50\,\Omega} + \frac{v_3}{100\,\Omega} = 4$$



Supernode is a kind of big node with 4 currents in this case…

Nodal analysis: Supernode with ground?



- Another example
- 20-V source is not the only element between nodes 1 and Ground (G)
- The branch current i_{20V} = (v_1 -20)/(2) is still available, KCL can still be applied to Node 1 → no need to use supernode!!!

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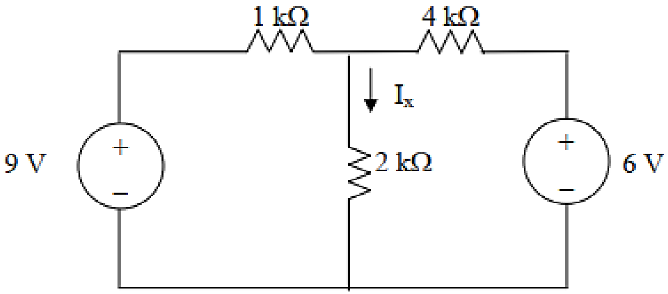
Nodal analysis: Your turn



Calculate **Ix** with nodal analysis.

Steps:

-Look at the circuit to identify essential nodes. How many here?

-Write the KCL equations for each essential node (not for the ground)

-Express the currents in the KCL equations as a function of nodal voltage

-Solve the system of equations -> in this problem then calculate Ix from the nodal voltage.

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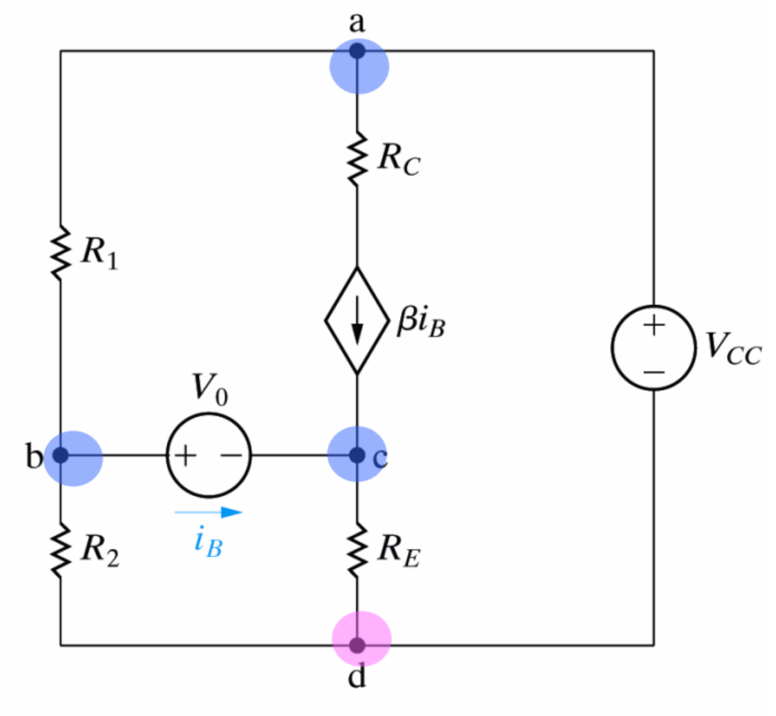
Example: Amplifier Circuit

- There are 3 unknown voltages.
- Since i_B cannot be derived by node voltages, we have 4 unknowns.
- The 2 voltage sources provide 2 constraints

$$\begin{cases} v_a = V_{CC} \\ v_c = v_b - V_0 \end{cases}$$



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Example: Amplifier Circuit

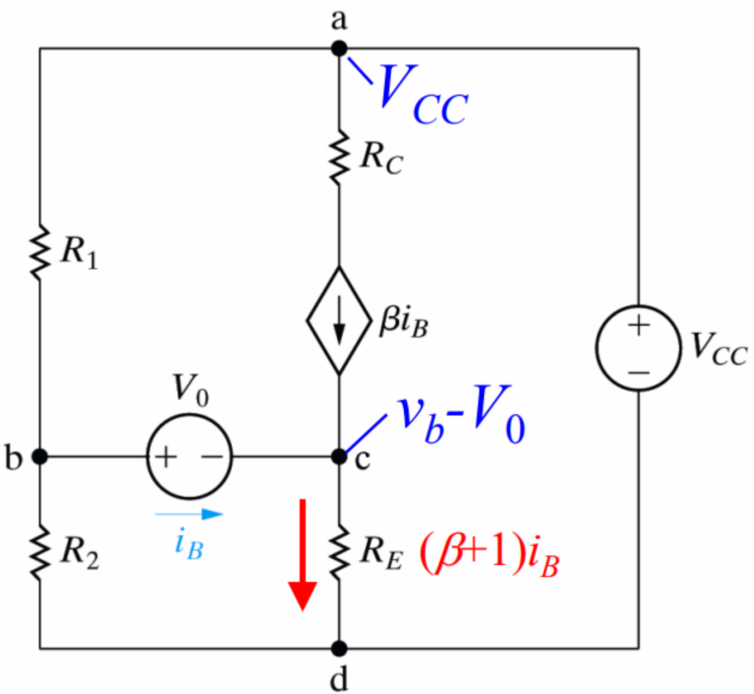
- Apply KCL to Node b:

$$\frac{v_b}{R_2} + \frac{v_b - V_{CC}}{R_1} + i_B = 0$$

- Apply KCL to Node c:

$$i_{cd} = (1+\beta)i_B,$$

$$\Rightarrow (1+\beta)i_B = \frac{v_c}{R_E}$$



This was just an example to demonstrate Nodal analysis to a circuit modelling a real amplifier. Nodal analysis is typically used by computer circuit simulators like Pspice.

Mesh Analysis

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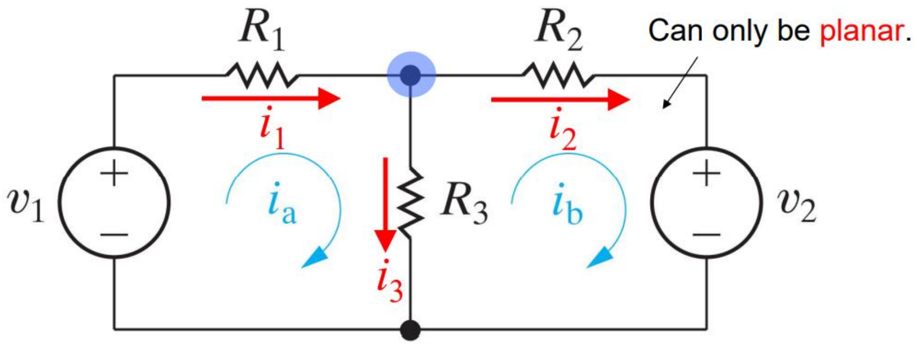
Mesh Analysis: Overview

- Comparable with the nodal analysis
- Makes use of Kirchhoff's **voltage** law
- But it is only applicable to planar circuits (no crossings)
- B-N+1 equations (N nodes, B branches)

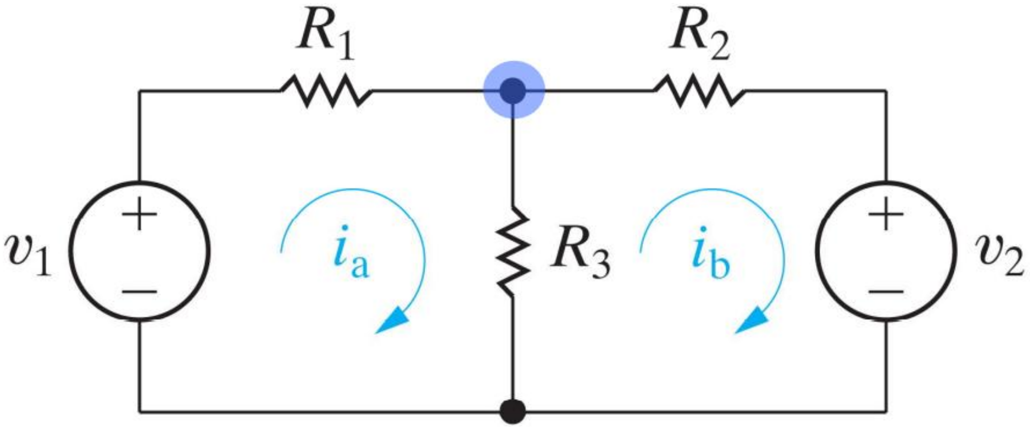
Mesh Analysis: Advantages

- Each mesh current flows into and out of any node on the way → automatically satisfy KCL
- Relation between branch currents and mesh currents: $i_1 = i_a$, $i_2 = i_b$, $i_3 = i_a - i_b$



Mesh Analysis: example (without numbers)

Start by drawing mesh currents and then write KVL equations



$$\begin{aligned}\text{Mesh a:} \quad & \begin{cases} v_1 = i_a R_1 + (i_a - i_b) R_3 \\ i_b R_2 + v_2 + (i_b - i_a) R_3 = 0 \end{cases} \\ \text{Mesh b:} \quad & \end{aligned} \Rightarrow \begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

*Tip: be mindful of the **polarity** of the voltages on the resistors (passive sign convention), especially if in between two meshes.*

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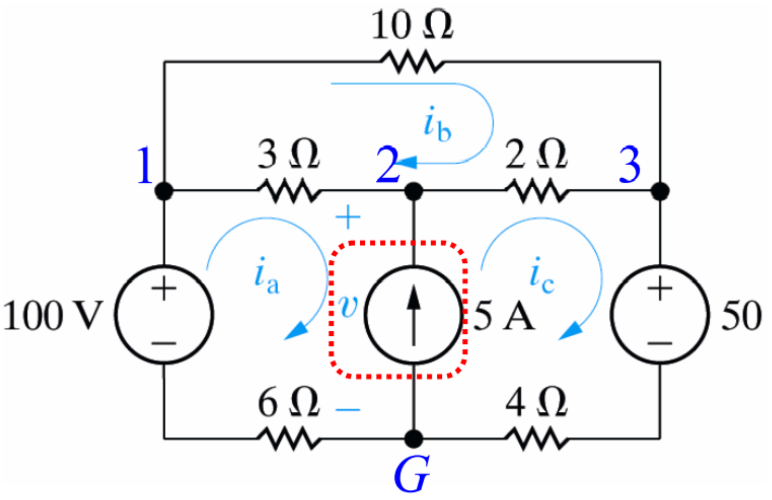
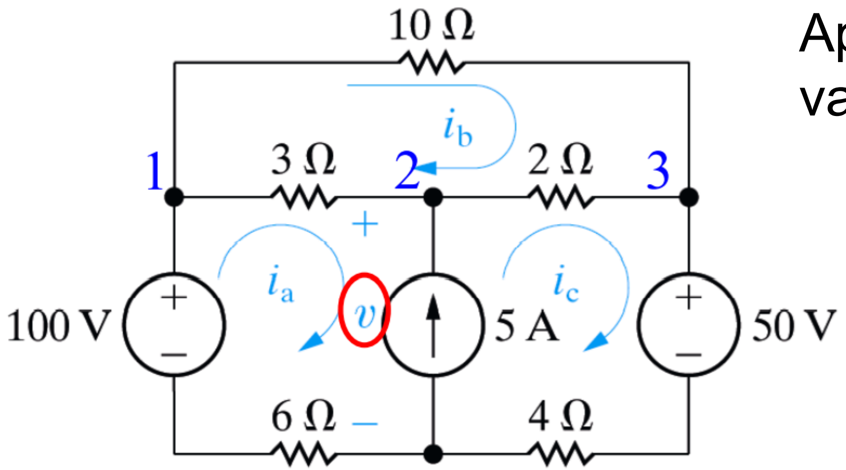
Mesh Analysis: current source between 2 meshes

- When a current source is between two essential nodes (no need to be the only element), the voltage drop across the source is undetermined, → cannot apply KVL to either mesh!



If the voltage at node 2 is not known (i.e. we cannot apply KVL) then we cannot write the mesh currents in a and c.

Mesh Analysis: current source between 2 meshes



Apply the elimination method to remove one variable and obtain the *1st equation*.

Mesh a: $(3\,\Omega)(i_a - i_b) + v + (6\,\Omega)i_a = 100,$

Mesh c: $(2\,\Omega)(i_c - i_b) + 50 + (4\,\Omega)i_c = v.$

$$\Rightarrow 9i_a - 5i_b + 6i_c = 50$$

The *2nd equation* comes from the top mesh with i_b:

$$10i_b = 3(i_a - i_b) + 2(i_c - i_b)$$

Then use the <u>constraint</u> coming from the current source in a KCL to get the *3rd equation*: $i_a + i_b + 5 = i_c$

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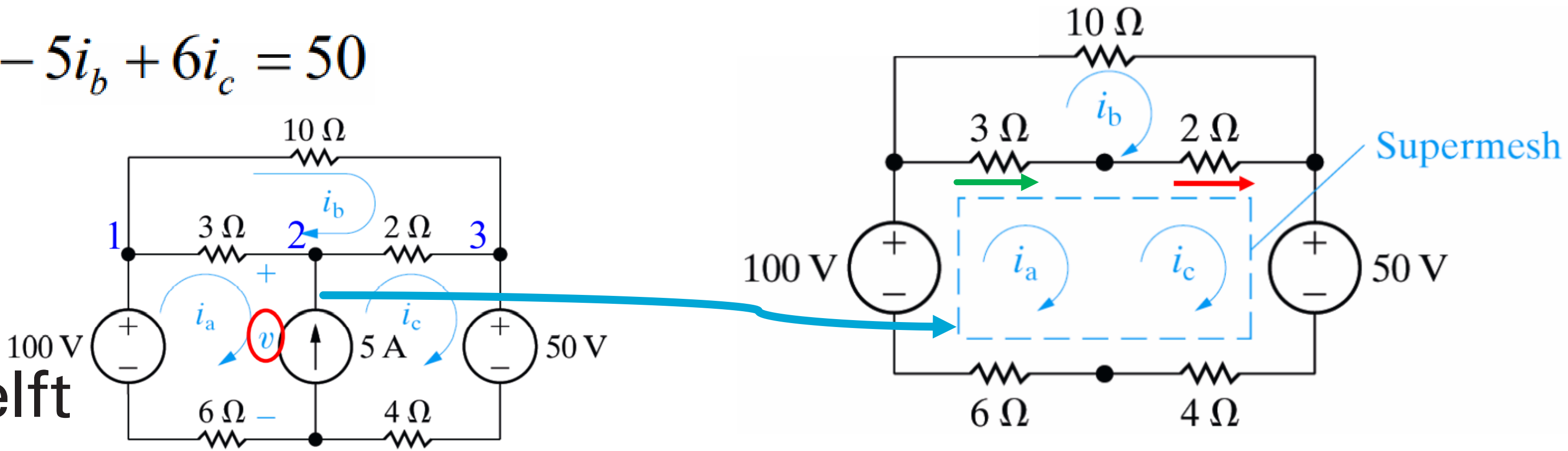
Mesh Analysis: current source between 2 meshes

- Alternative approach: apply KVL to a **supermesh** formed by the combination of two meshes; one can get the same equation as in the previous slide without the intermediate elimination step. The other two equations are derived as in the previous slide.

$$(3\,\Omega)(i_a - i_b) + (2\,\Omega)(i_c - i_b) + 50 + (4\,\Omega)i_c + (6\,\Omega)i_a = 100$$

$$\Rightarrow 9i_a - 5i_b + 6i_c = 50$$



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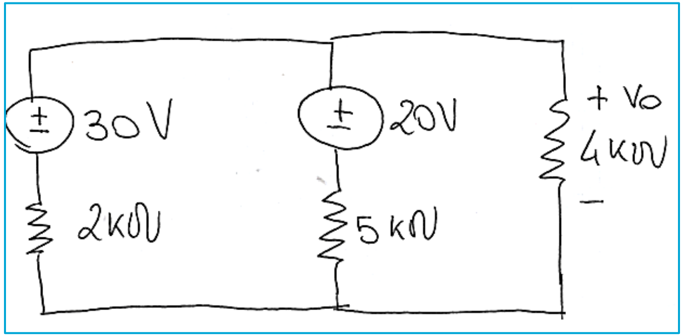
Mesh analysis: Your turn

Calculate **Vo** with mesh analysis.



Steps:

-No panic (always!)

-Look at the circuit to identify meshes and write the mesh currents

-Write the KVL equations for each mesh

-Express the voltages in the KVL equations as a function of mesh currents

-Solve the system of equations then find Vo using Ohm's law.

Expected result V_o=20V

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Coffee Break

Suitability of Each Method

Suitability of Each Method

- In principle both the nodal analysis and mesh analysis are useful for any given circuit.
- What then determines if one is going to be more efficient for solving a circuit problem?
- There are two factors that dictate the best choice:
  - The topology of the particular network is the first factor.
  - The second factor is the information required (i.e. what the problem asks for).

Suitability of Each Method

Mesh analysis:

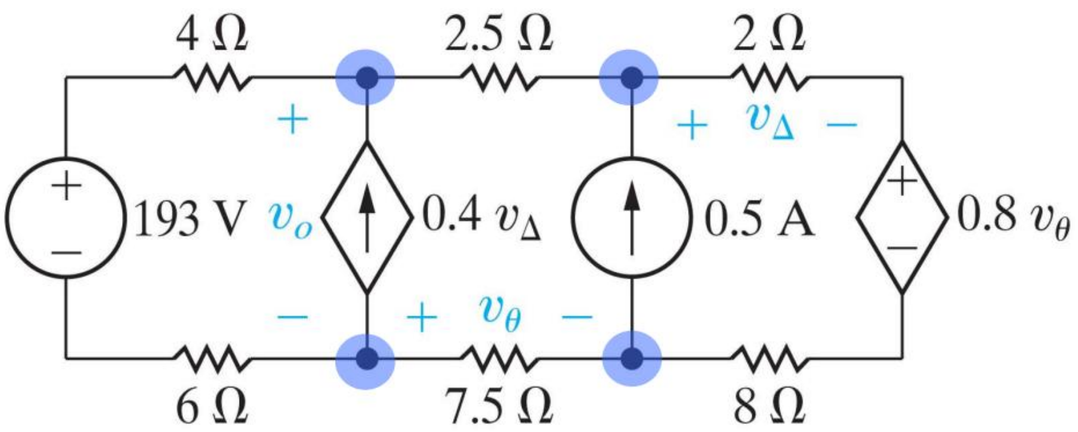
- If the network contains:
  - Many series connected elements.
  - Voltage sources.
  - Supermeshes.
  - A circuit with fewer meshes than nodes.
- If branch/mesh currents are the unknown quantity

Nodal analysis:

- If the network contains:
  - Many parallel connected elements.
  - Current sources.
  - Supernodes.
  - A circuit with fewer nodes than meshes.
- If nodal voltages are the unknown quantity

LOOK at your circuit and the question(s) before writing equations!

Example: Suitability of Each Method



- 3 meshes, 2 current sources (2 supermeshes) → 1 mesh equation.
- 4 essential nodes, no voltage source is the only element on one branch (no supernode) → (4-1)= 3 node equations

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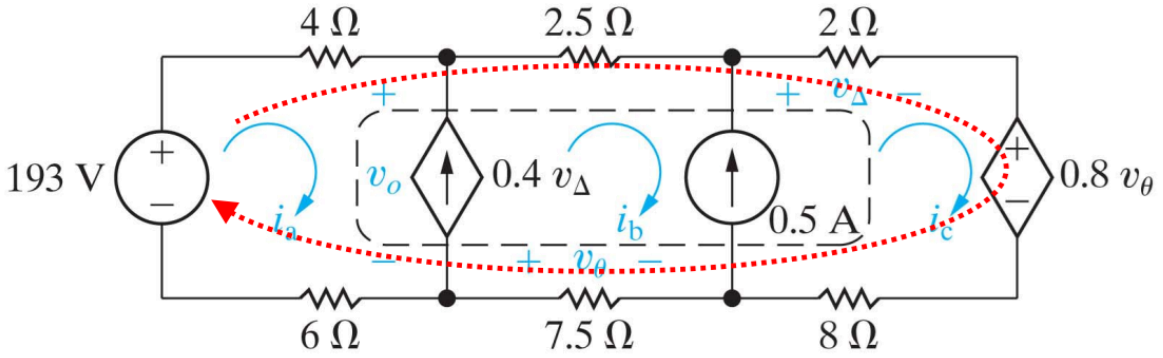
Example: Suitability of Each Method



- Apply KVL to supermesh a, b, c

  $$193 = 4i_a + 2.5i_b + 2i_c + 0.8v_\theta + 8i_c + 7.5i_b + 6i_a$$

- But note that $v_\theta = -7.5i_b$ (do not forget the minus!)
- So eventually: $\mathbf{193 = 10}\boldsymbol{i_a} + \mathbf{4}\boldsymbol{i_b} + \mathbf{10}\boldsymbol{i_c}$

But you still need two more equations…

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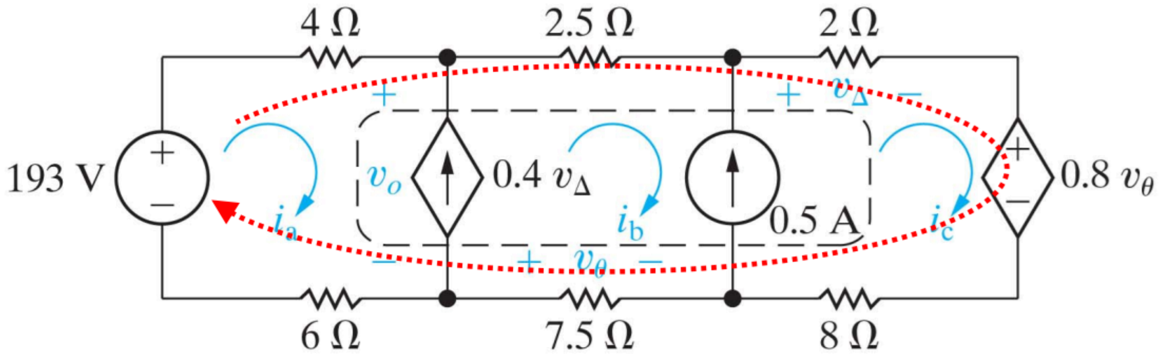
Example: Suitability of Each Method



- You get the 2 equations from the constraints of the current sources of the supermesh, i.e. KCL…

$$\mathbf{0.5 + i_b = i_c}$$

And $i_b = i_a + 0.4v_\Delta = i_a + 0.4(2i_c) \rightarrow \boldsymbol{i_b = i_a + 0.8i_c}$

Now you are ready to go and solve the system of 3 equations.

Feel free to practice when you revise with this exercise!

Summary

- Methods for circuit analysis
  - Visual Inspection
  - Nodal analysis (based on KCL with nodal voltages as unknowns)
  - Mesh analysis (based on KVL with mesh currents as unknowns)

- Suitability of each method -> it comes with practical experience, **so practice, practice, practice!!!**

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Next steps

- **SGH** (Self-Graded Homework assignments): posted today; submission due on Wednesday.

- **Seminar**: in groups on Tuesday & altogether on Friday.

- **Next week**:
  - Linearity
  - Superposition
  - Norton and Thévenin theorems & equivalent networks
  - Maximum power transfer

  - **1hour DRY-RUN EXAM** on Tue September 23rd 16-17 (do take advantage of this!)

Thank you!

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