EE1C1 “Linear Circuits A”

Week 1.4

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Faculty of Electrical Engineering, Mathematics and Computer Science

Today

- Recap W: Nodal and mesh analyses
- Week 1.4:
  - Linearity, superposition
  - Source Transformation
  - Thévenin and Norton Theorems
  - Maximum power transfer
- Summary and Next Week

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Recap

Nodal and Mesh Analysis

Mesh analysis:

Goal:

- Solve the unknown mesh currents using Kirchhoff's voltage law (KVL)

Procedure:

- Define the mesh currents in the circuit and name them, e.g. I_1, I_2, I_3 etc..
- Use KVL for all meshes and express everything in terms of mesh currents
- Solve the system of equations for the unknown mesh currents

Nodal analysis:

Goal:

- Solve the unknown node voltages using Kirchhoff's current law (KCL)

Procedure:

- Select a reference (ground) node
- Give all the other nodes a name e.g. V_1, V_2, V_3 etc..
- Use KCL at all nodes (except for ground) and express everything in terms of node voltages
- Solve the system of equations for the unknown node voltages

Suitability of Each Method

Mesh analysis:

- If the network contains:
  - Many series connected elements.
  - Voltage sources.
  - Supermeshes.
  - A circuit with fewer meshes than nodes.
- If branch/mesh currents are the unknown quantity

Nodal analysis:

- If the network contains:
  - Many parallel connected elements.
  - Current sources.
  - Supernodes.
  - A circuit with fewer nodes than meshes.
- If nodal voltages are the unknown quantity

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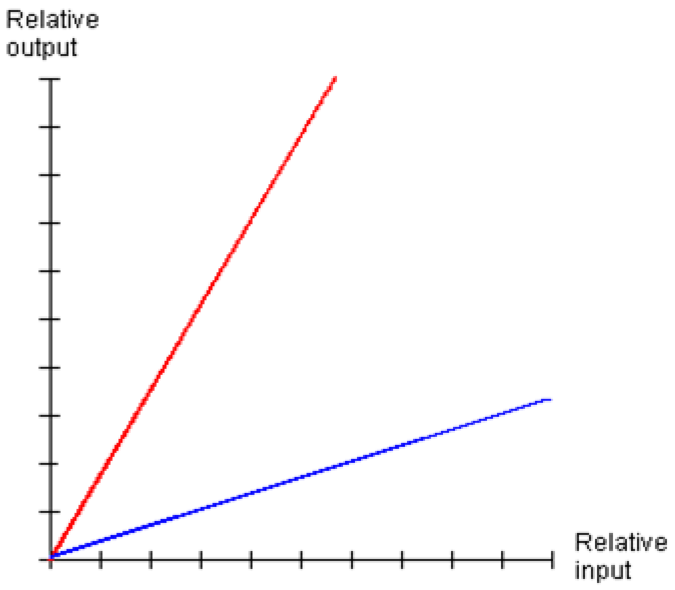
LOOK at your circuit and the question(s) before writing equations!

Week 1.4

Our circuits are linear

- Modelled by means of a system of linear equations
- The transfer from individual sources to the outputs is linear:
  - N× input yields N× output
  - A straight line through the origin on an input-output plane



Examples of linearity

Superposition

- “The currents and voltages in any linear circuit with independent sources can be evaluated as the algebraic **sum of the contributions from each source**, independently.”

- Useful in designing a large system, where the impact of each independent source is critical for system optimization. *It allows you to solve multiple but much simpler circuits compared to one complex circuit*.

Superposition Recipe

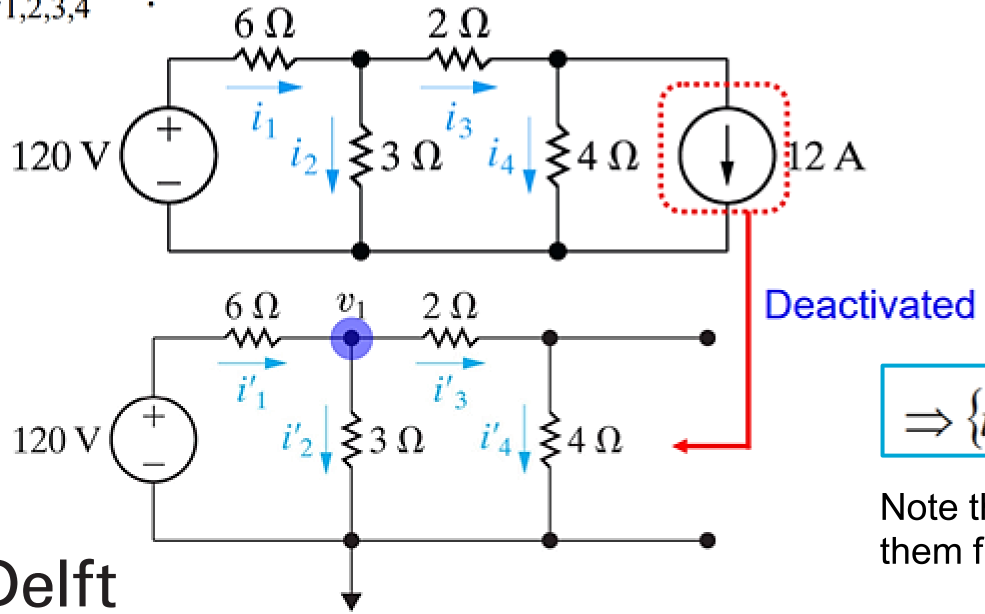
1. Take one source, turn all other off
2. Replace voltage sources by short circuits
3. Replace current sources by open circuits

4. Evaluate the requested voltages / currents
5. Add all partial results

Example of superposition (2 sources)

This circuit has two independent sources. We first deactivate the current one. The remaining circuit can be solved by nodal analysis (note the two resistor in series to simplify your life)

$i_{1,2,3,4} = ?$



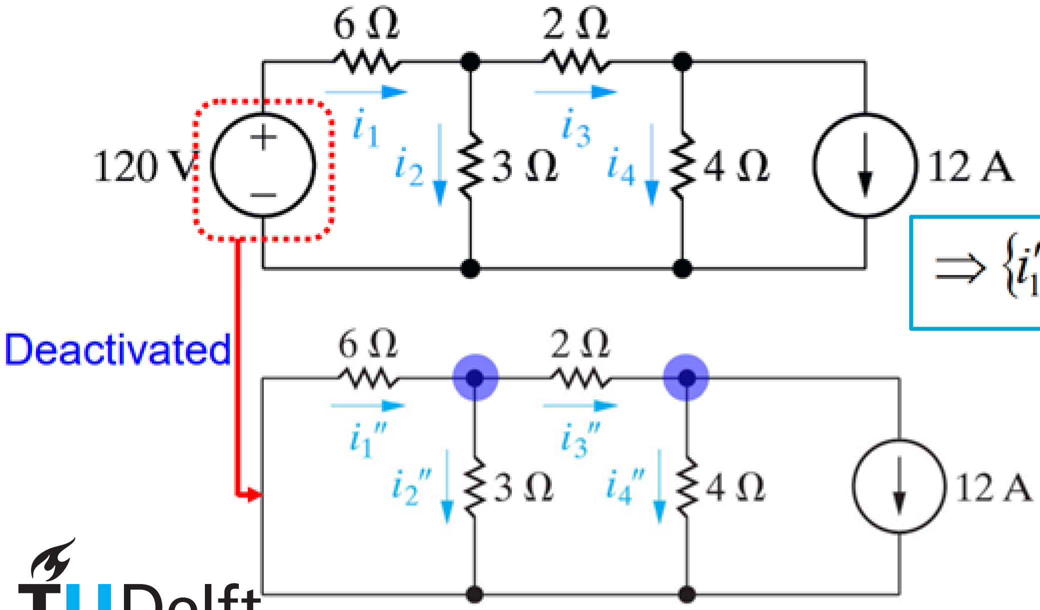
$$\frac{v_1}{3} + \frac{v_1}{(2+4)} = \frac{120 - v_1}{6}$$

$$\Rightarrow \{i'_1 = 15\ \text{A},\ i'_2 = 10\ \text{A},\ i'_3 = i'_4 = 5\ \text{A}\}.$$

Note the prime sign on the currents to differentiate them from the currents in the original circuit!

Example of superposition (2 sources)

Then we deactivate the voltage source.
This can be still solved by nodal analysis.



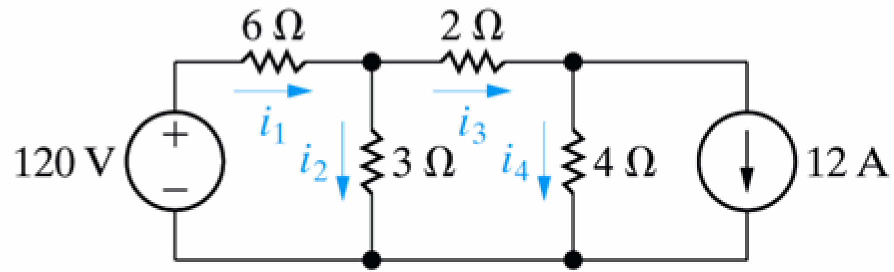
$$\begin{cases} -\dfrac{v_1}{6} = \dfrac{v_1}{3} + \dfrac{v_1 - v_2}{2} \\ \dfrac{v_1 - v_2}{2} = \dfrac{v_2}{4} + 12 \end{cases}$$

$$\Rightarrow \{i_1'' = 2\text{ A},\ i_2'' = -4\text{ A},\ i_3'' = 6\text{ A},\ i_4'' = -6\text{ A}\}.$$

Note the double prime sign on the currents to differentiate them from the currents in the original circuit!

Example of superposition (2 sources)

Then, you sum the partial results together to obtain the final results.
The final results are the superposition (sum) of the partial outputs of each separate input.



$$\Rightarrow \begin{cases} i_1 = i_1' + i_1'' = 15 + 2 = 17\text{ A}, \\ i_2 = i_2' + i_2'' = 10 - 4 = 6\text{ A}, \\ i_3 = i_3' + i_3'' = 5 + 6 = 11\text{ A}, \\ i_4 = i_4' + i_4'' = 5 - 6 = -1\text{ A}, \end{cases}.$$

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Remarks on Superposition

Superposition:

- is a fundamental property of linear equations
- CAN be applied to currents and voltages
- CANNOT be applied to powers
- **Why not?**

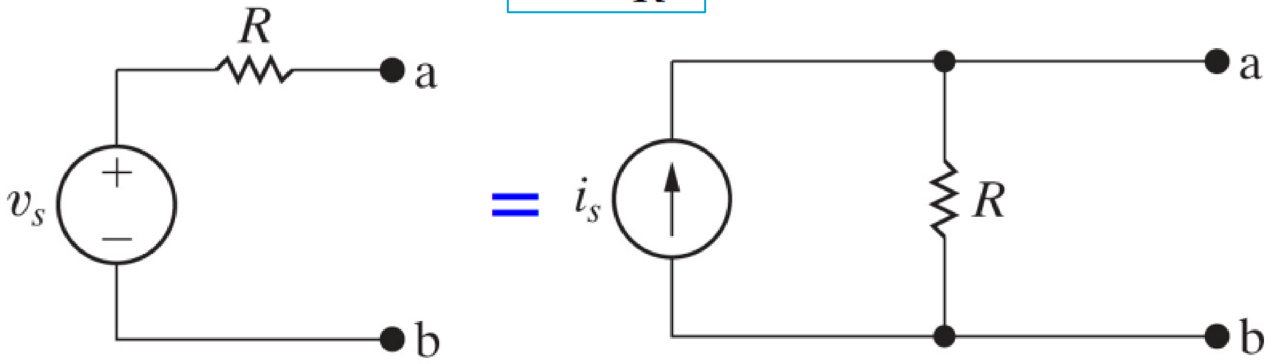
  → Not a linear expression (multiplication)

Source Transformation

Source Transformation

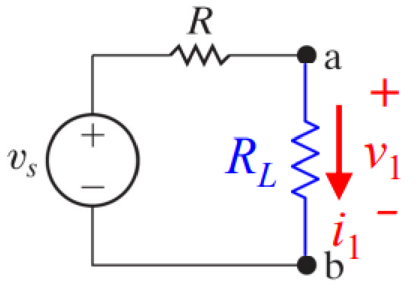
- A voltage source v_s in series with a resistor R can be replaced by a current source is in parallel with the same resistor R or vice versa, where:

$$i_s = \frac{v_s}{R}$$

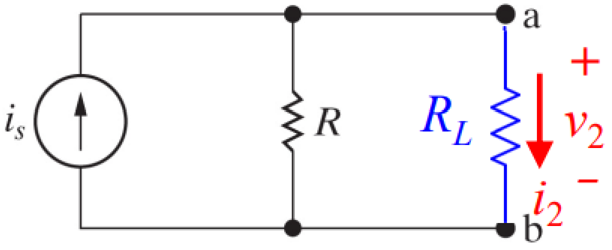


Source Transformation: Proof

- For any load resistor R_L , the current i and voltage v between terminals a-b should be consistent, equal, in both configurations. The proof comes from application of voltage/current dividers.



$$i_1 = \frac{v_s}{R+R_L}, \quad v_1 = \frac{R_L}{R+R_L} v_s .$$



$$\begin{cases} i_2 = \dfrac{R}{R+R_L} i_s = \dfrac{R}{R+R_L} \dfrac{v_s}{R} = i_1, \\ v_2 = i_s (R // R_L) = \dfrac{v_s}{R} \dfrac{R R_L}{R+R_L} = v_1 . \end{cases}$$

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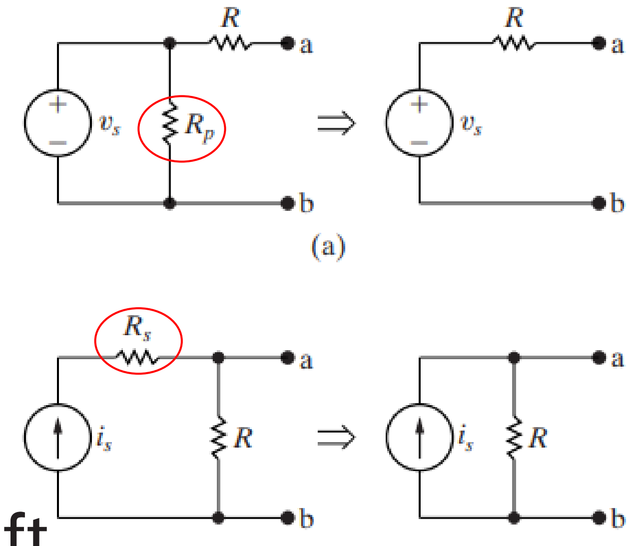
Redundant Resistors

- Redundant = resistors that have no effect on the equivalent circuit that predicts the behavior with respect to terminals a-b. These are **resistor in parallel to a voltage source and in series to a current source.**



(a)

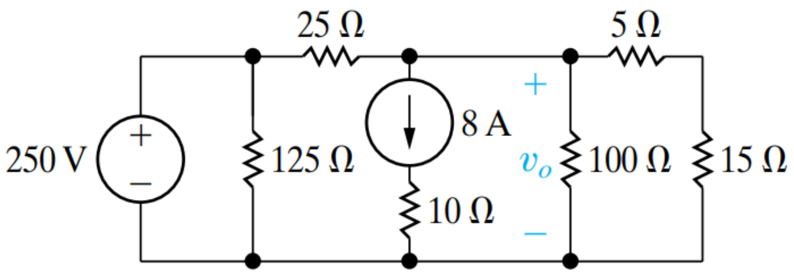
(b)

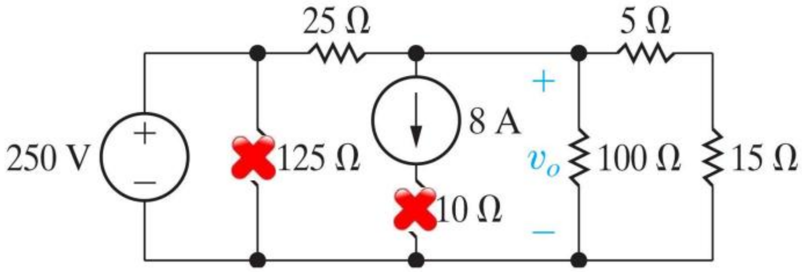
The two circuits depicted on the left are equivalent to those on the right with respect to terminals a-b because they produce the same voltage.

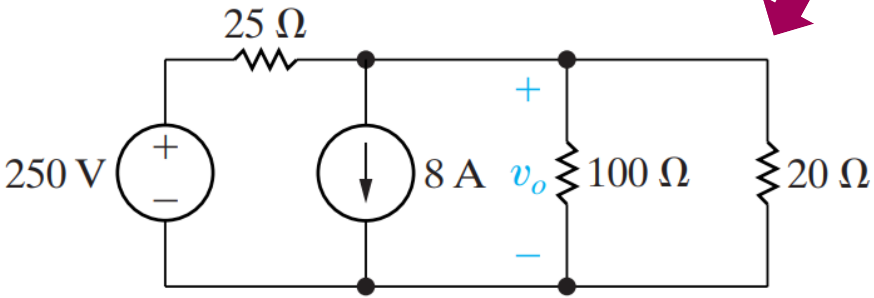
Source Transformation Example

Find the voltage v_0 and the power supplied by the 250V source.

Note that if you start writing equations upfront you have many equations, so it is worth simplifying first.



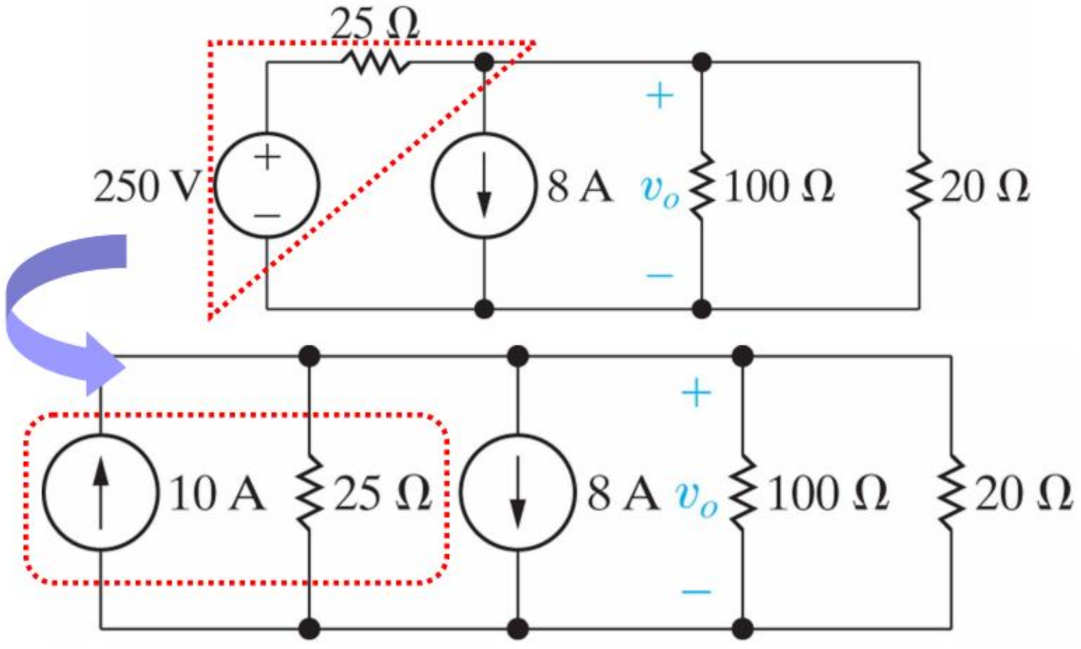




Removed redundant resistors & note that 5 and 15 Ohm are in series

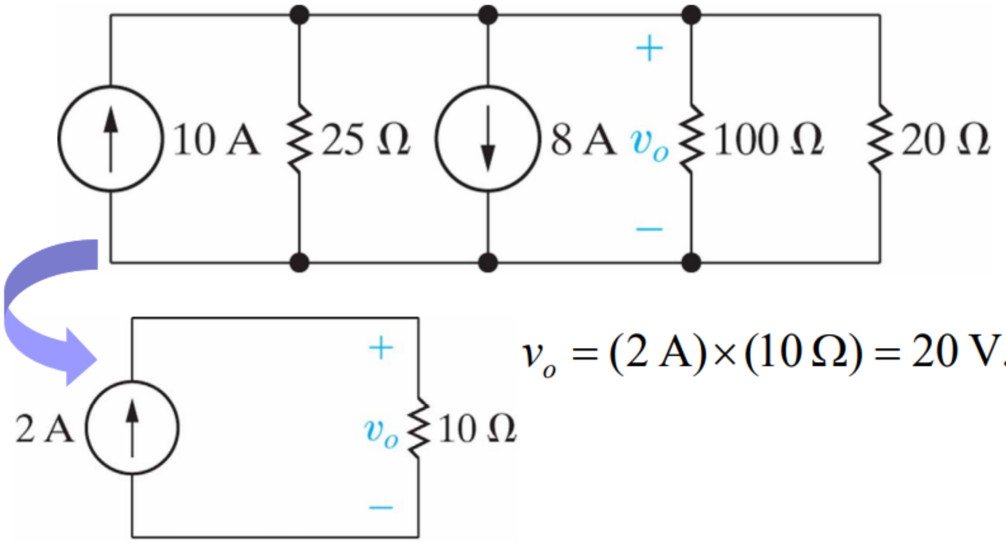
Source Transformation Example Continued

Transform the voltage source into a current source



Source Transformation Example Continued

…and simplify the circuit as now all the components are in parallel.

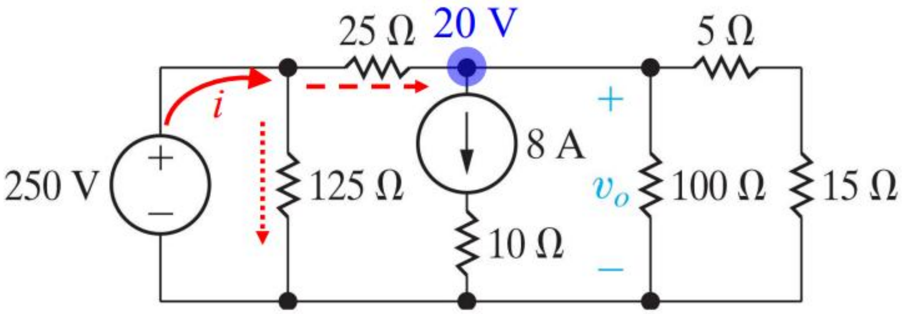


$$v_o = (2\ \text{A}) \times (10\ \Omega) = 20\ \text{V}.$$

Source Transformation Example Continued

Suppose that you were asked to compute the power on the voltage source.

This must be calculated based on the original circuit.



$$i = \frac{250\text{ V}}{125\,\Omega} + \frac{(250-20)\text{ V}}{25\,\Omega} = 11.2\text{ A},$$

$$P_{250\text{V}} = -(11.2\text{ A}) \times (250\text{ V}) = -2.8\text{ kW}.$$

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Thévenin & Norton Theorems

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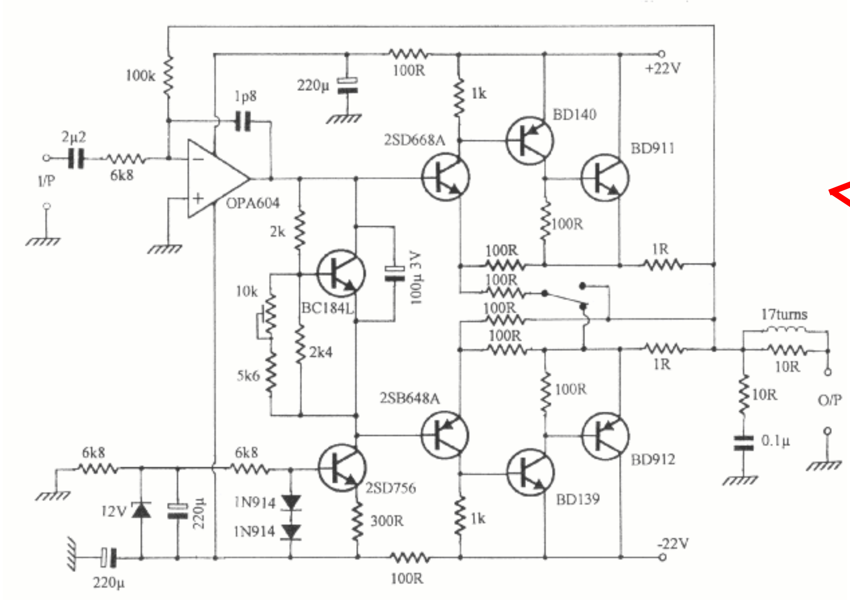
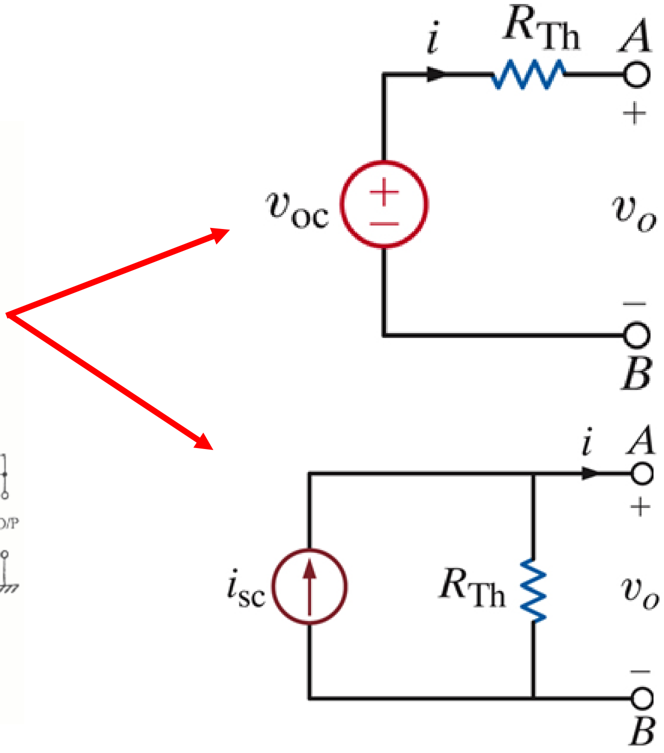
Thévenin & Norton Theorems

Low distortion audio power amplifier





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Very powerful tools: they transform a circuit, however complex, into an equivalent one made by one source plus one resistor!

Thévenin & Norton Theorems

Hermann von Helmholtz
1821-1894

Léon Charles Thévenin
1857-1926

Hans Ferdinand Mayer
1895-1980

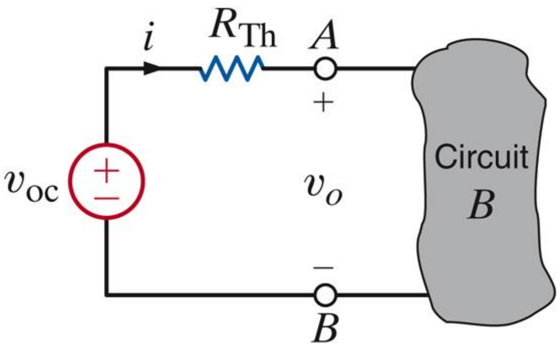
Edward Lawry Norton
1898-1983

- D. H. Johnson, "Origins of the equivalent circuit concept: The voltage-source equivalent," *Proc. IEEE*, vol. 91, no. 4, pp. 636-640, Apr. 2003.
- D. H. Johnson, "Origins of the equivalent circuit concept: The current-source equivalent," *Proc. IEEE*, vol. 91, no. 5, pp. 817-821, May 2003.

Key engineers and publications to know more…

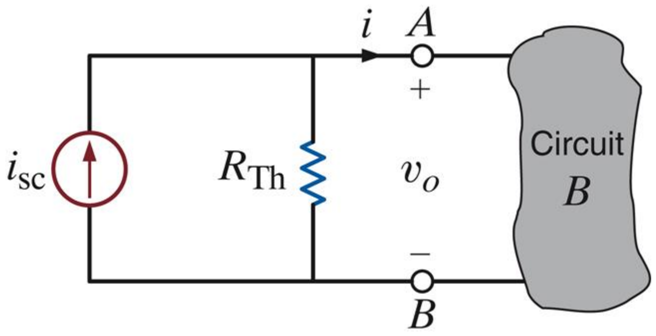
Thévenin Theorem

- A linear circuit is connected to a load (can be one component, or an entire circuit, Circuit B here)
- The original linear circuit can be **replaced by an equivalent circuit** consisting of one independent source in series with one equivalent resistance
- The circuit's overall behaviour does not change.



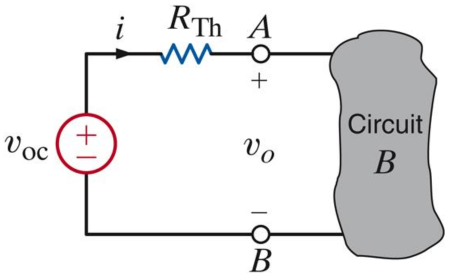
Norton Theorem

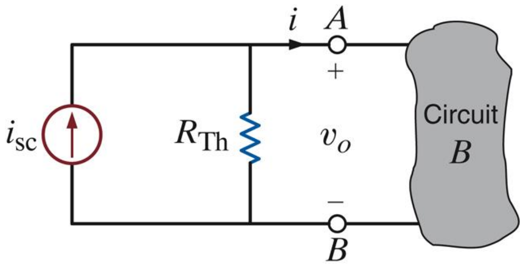
Similar to Thévenin, but in this case the linear circuit is replaced by one independent current source in parallel with one equivalent resistance.



Application

- OK, then the derivation of the equivalent circuits implies computing the values of the sources and the equivalent resistors.
- Approaches vary depending on what sources the original circuit contains:
  1. **with independent sources, only**
     - determine V_{oc} or I_{sc}; determine R_{Th}
  2. **with dependent sources, only**
     - V_{oc} and I_{sc} = 0; determine only R_{Th} by connecting a “test-source”
  3. **both independent and dependent sources** -> a bit more work





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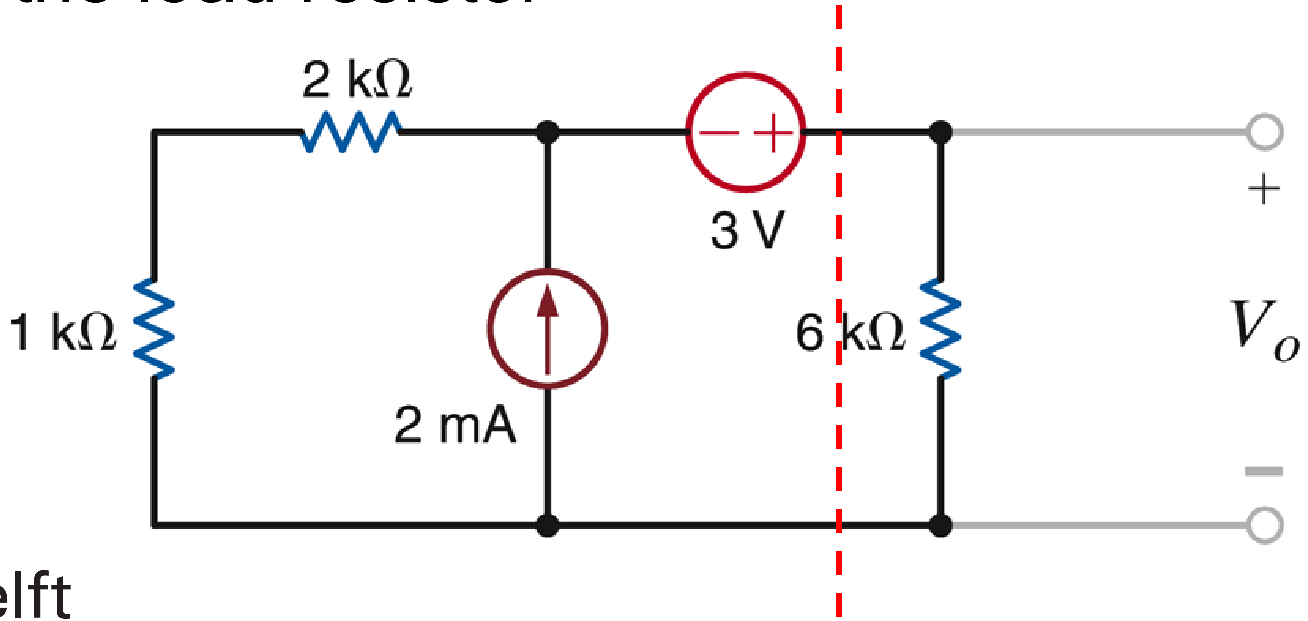
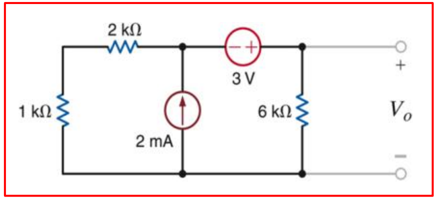
T&N: Independent Sources Only

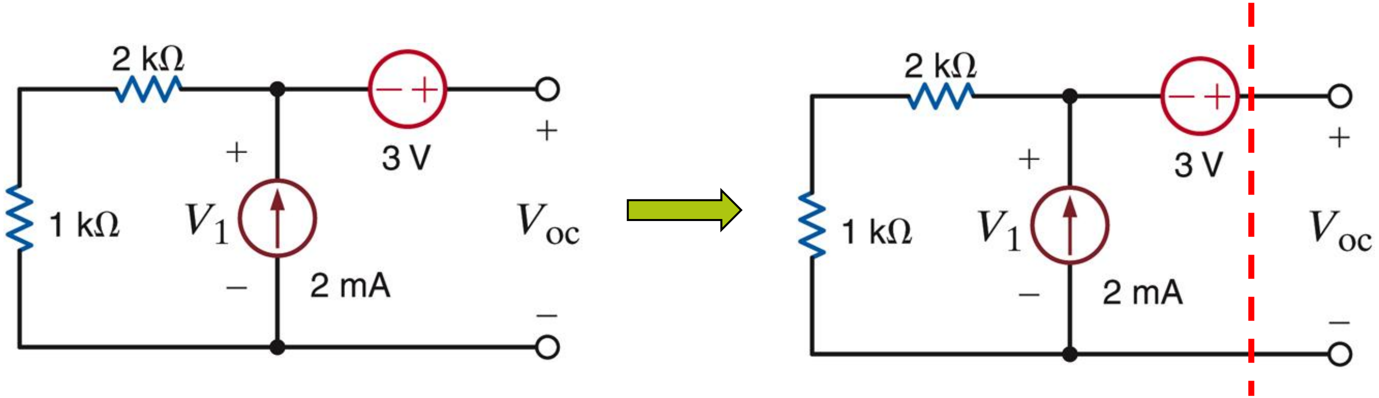
- We employ Thévenin for determining V_o on the load resistor



T&N: Independent Sources Only



- We examine the circuit from the 6 kΩ load
- Determine V_{oc} (OC stands for open circuit)



- V_1 = 6 V (simple inspection of the mesh on the left-hand side)
- → **V_{oc} = 9 V** (KVL on the right-hand side of the circuit)

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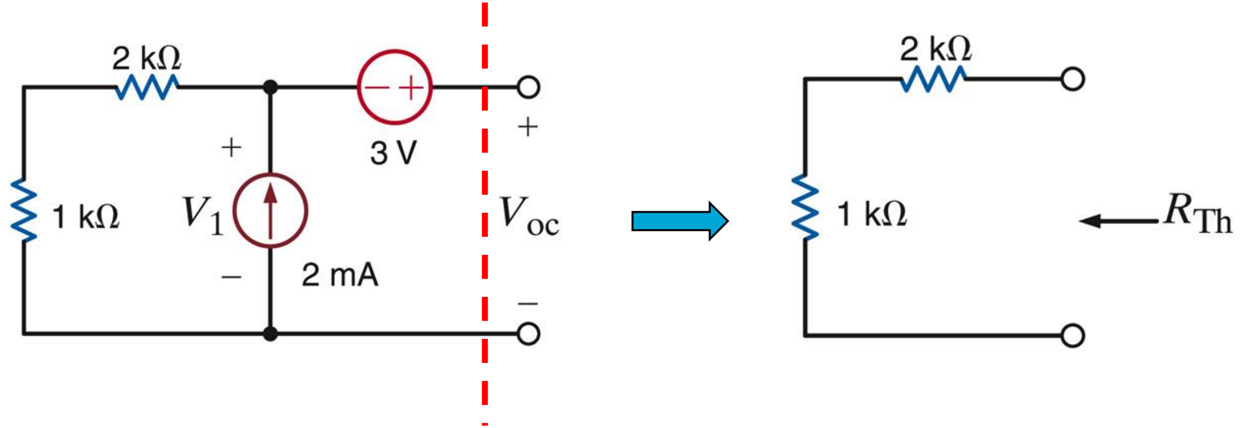
T&N: Independent Sources Only

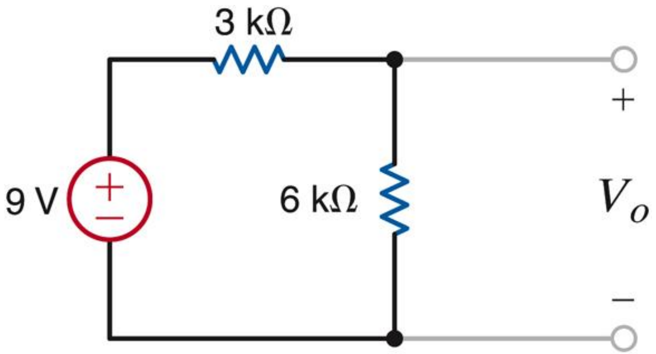
- Determine R_{Th} – For this step, all independent sources are turned off
  - a current source is replaced by an open circuit
  - a voltage source is replaced by a short circuit



- Not so difficult: $\boldsymbol{R_{Th}}$ = 2kΩ + 1kΩ = **3kΩ** - Here the trick is to start combining resistors from the opposite side of the terminals, and work backwards…

T&N: Independent Sources Only

- To conclude with, the Thévenin equivalent is connected to the original load. Always redraw the equivalent circuit for clarity.



- It follows that $\boldsymbol{V_o = 6\ V}$ which is the quantity we were asked for at the beginning of the exercise.

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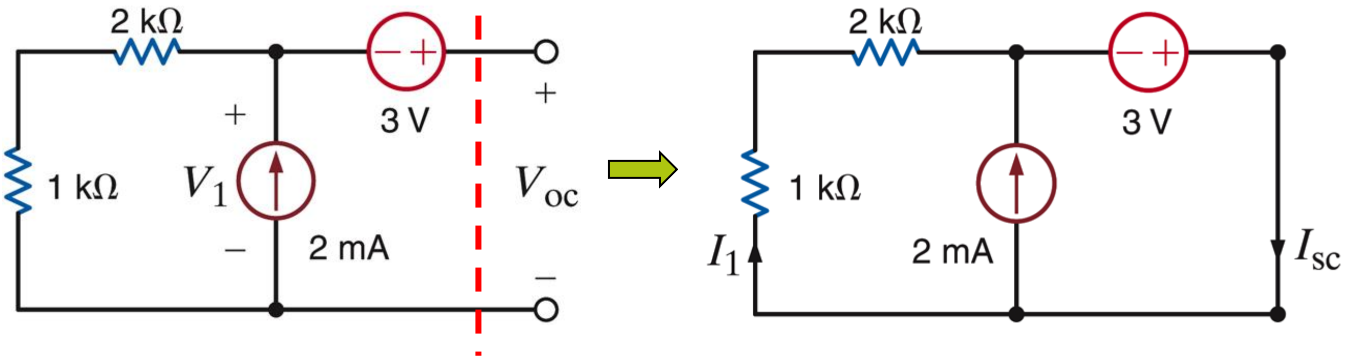
T&N: Independent Sources Only

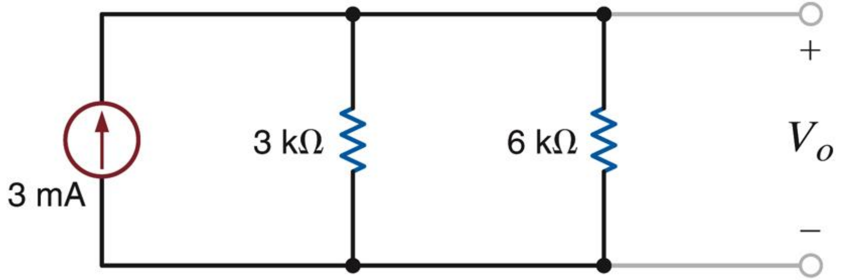
And what about Norton?



- For Norton we look for the short-circuit current (SC = short circuit)
- The short circuit transfers the 3V source on the two resistances
- This yields an additional 1mA current (I_1=3V/3kΩ=1mA)
- Summarising: $\mathbf{I_{sc}}$ is 1 mA + 2 mA = **3 mA**

T&N: Independent Sources Only

- R_{Th} is already determined (it is the same as in Thévenin)
- Now we connect the Norton equivalent to the original load

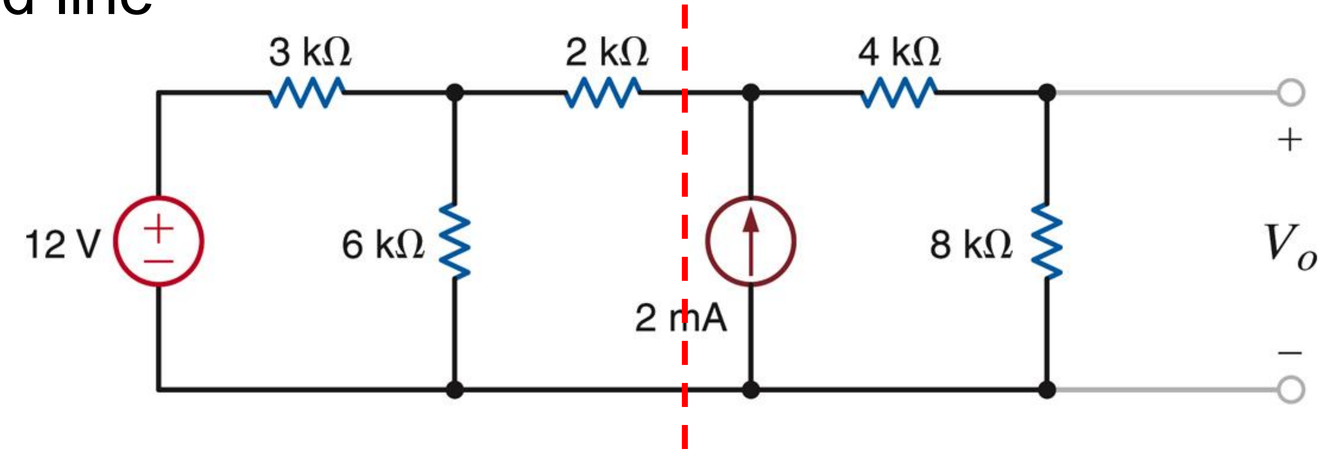


- V_o is again **6V** as expected

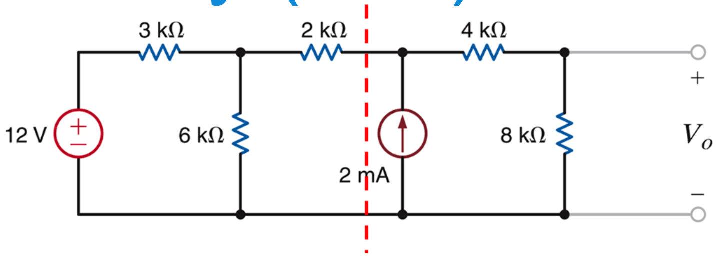
T&N: Independent Sources Only (Ex2)

- We employ Thévenin for determining V_o

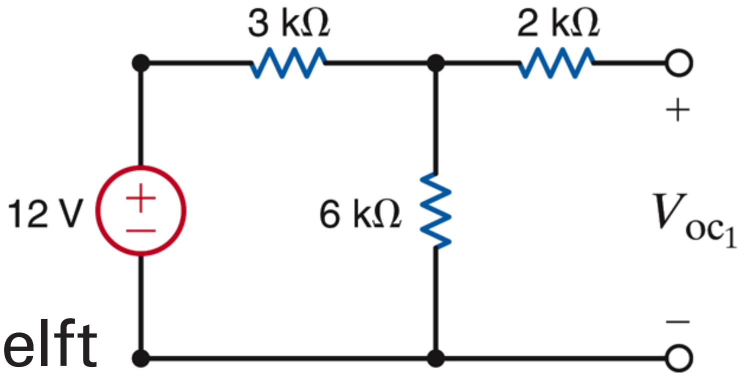
Note that we transform the circuit to the left of the red dashed line



T&N: Independent Sources Only (Ex2)

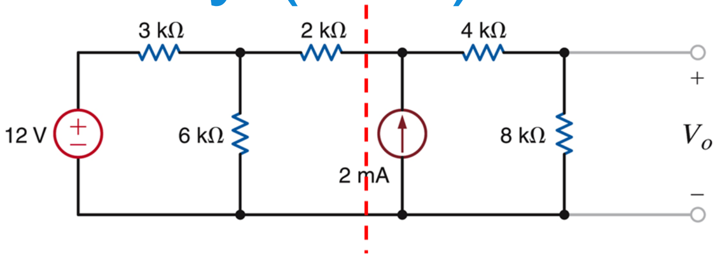


We consider firstly the network on the left of the current source and determine the open circuit voltage. Note that this circuit is basically a simple voltage divider (the 2kΩ resistor has no voltage drop!).

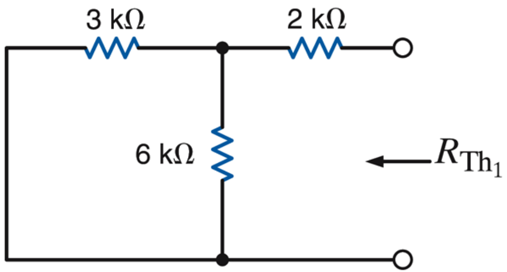


$$V_{oc1} = 12\text{V}\left(\frac{6\text{k}\Omega}{6\text{k}\Omega + 3\text{k}\Omega}\right) = 8\,\text{V}$$

T&N: Independent Sources Only (Ex2)



- Then we determine the Thévenin resistance
  - The voltage source becomes a short circuit
  - Make sure it's clear to you how the resistors combine



$$R_{Th_1} = 2k\Omega + \frac{(3k\Omega)(6k\Omega)}{3k\Omega + 6k\Omega} = 4k\Omega$$

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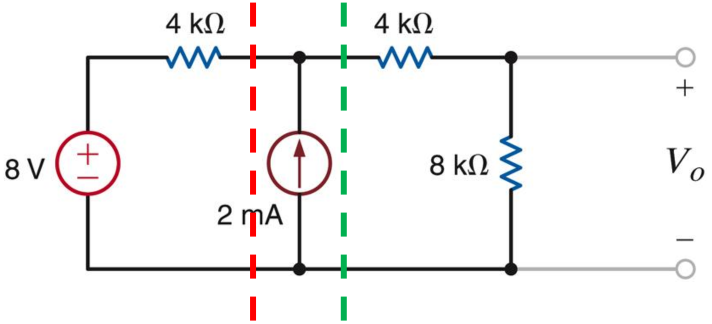
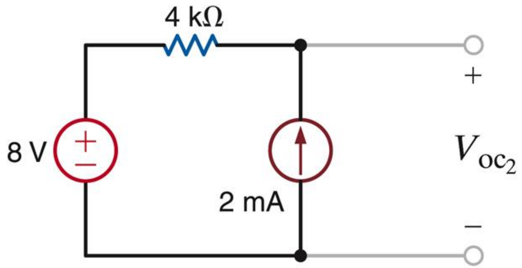
T&N: Independent Sources Only (Ex2)

- We add the Thévenin equivalent to the right-hand side of the original circuit



- And we apply Thévenin again, this time to the left of the green dashed line



$$V_{oc_2} = (2\text{mA})(4\text{k}\Omega) + 8\text{V} = 16\,\text{V}$$

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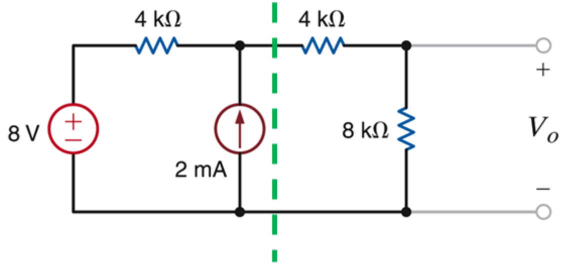
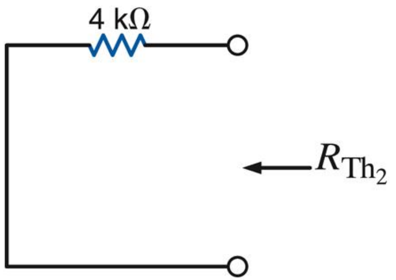
T&N: Independent Sources Only (Ex2)

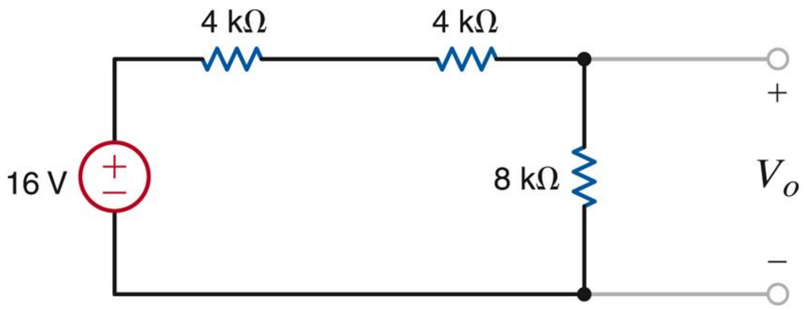


- We again determine the Thévenin resistance



- R_{Th2} = 4kΩ

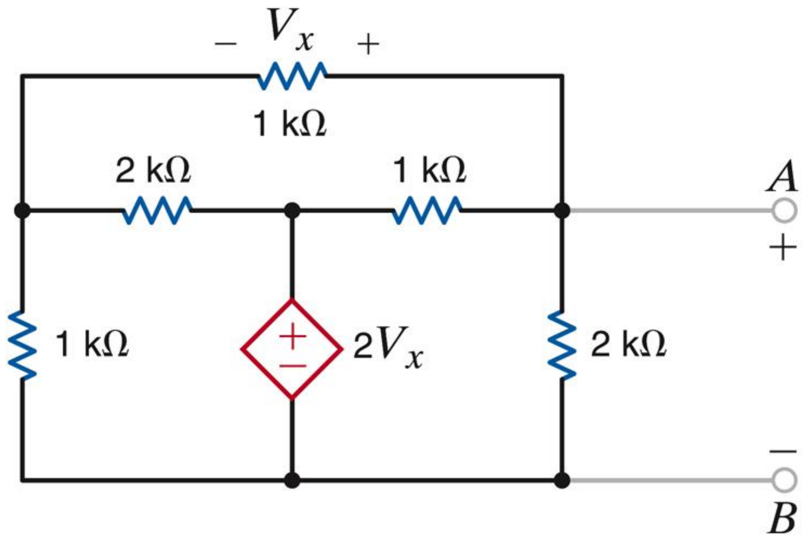
- The solution is computed via voltage divider…



Coffee Break

T&N: Dependent Sources Only

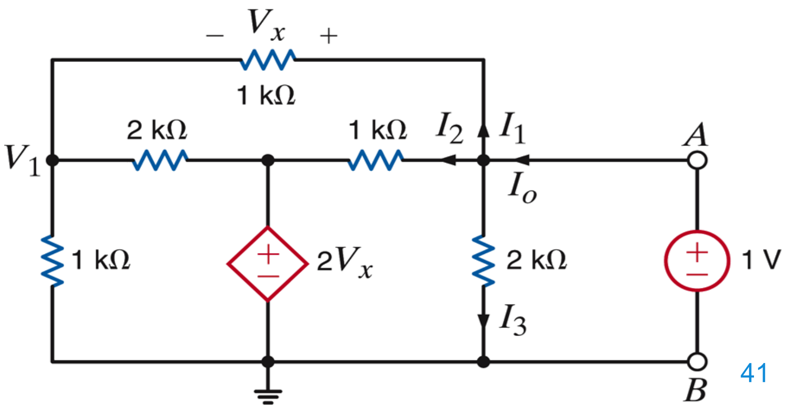
- In a circuit with dependent sources only, all open-circuit voltages and short-circuit currents **are null.** The equivalent circuit therefore **is just a resistor!**



T&N: Dependent Sources Only

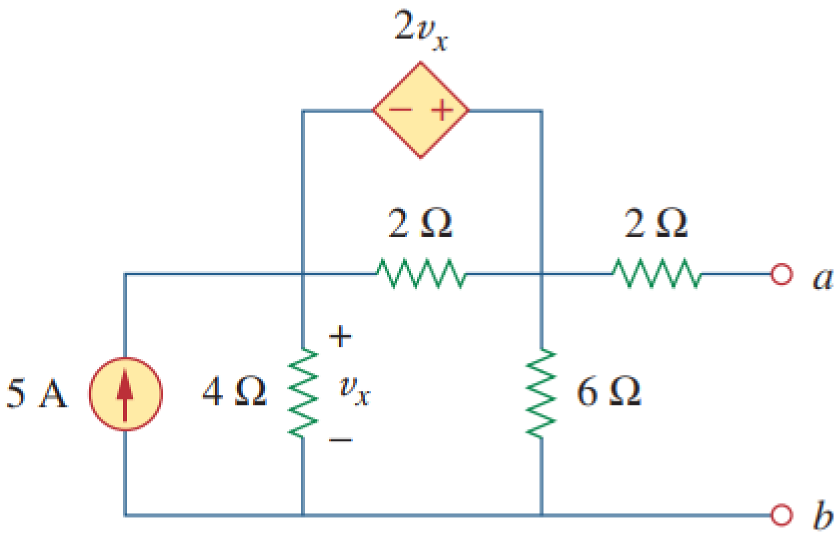
- No independent voltage or current source -> Yields only an R_{Th}
- We connect a "***test voltage source***" of 1 V and we determine I_o – Then the equivalent resistance is the ratio of the test voltage source divided by the current I_o i.e. $R_{th}=1/I_o$
- One can also connect a "***test current source***" of 1 A and compute the voltage across this V_o to then compute the equivalent resistance as $R_{th}=V_o/1$

Note that the equivalent resistance in this case can be *negative*. This implies that the circuit is supplying power to the load (see the corresponding example in the book).



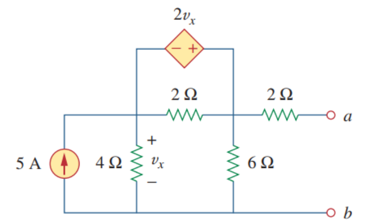
T&N: Dependent and Independent Sources together

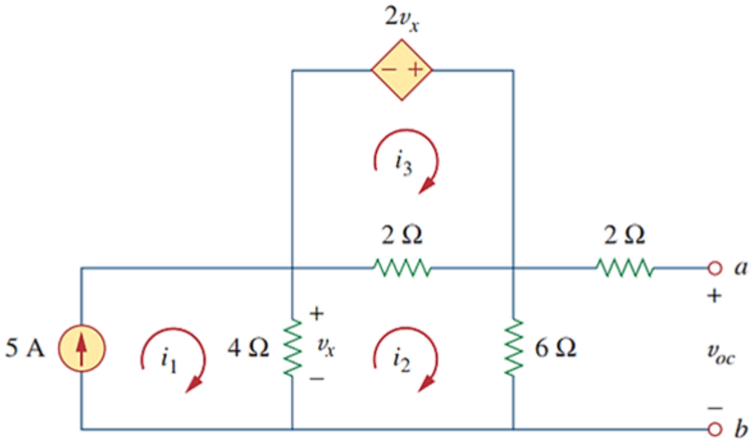
- Find the Thevenin equivalent of the circuit below at terminals a-b



T&N: Dependent and Independent Sources together

To **find V_{oc}** we solve the below circuit. For example, we can use mesh analysis:





$$i_1 = 5$$

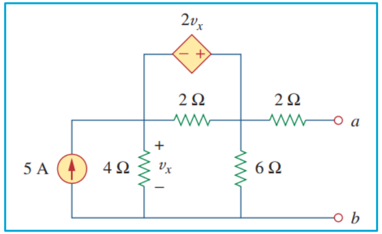
$$-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

Solving the system leads to i_2=10/3 A

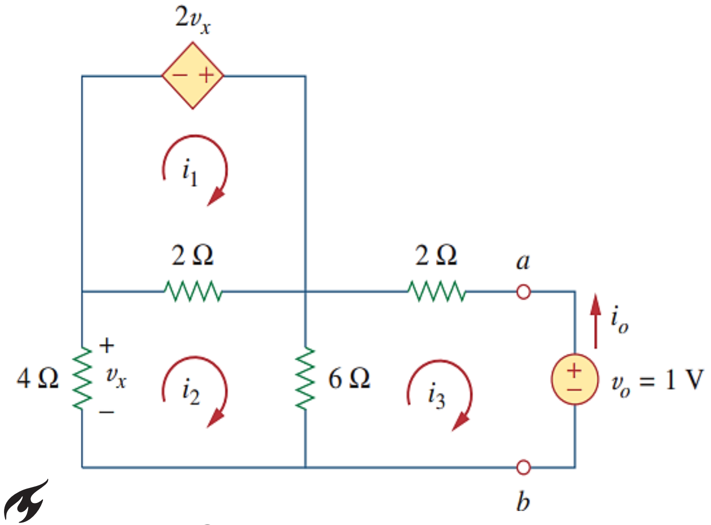
V_{oc} is then $6i_2$, hence **20V**

T&N: Dependent and Independent Sources together



Now R_{th}. First, we need <u>BOTH</u> to deactivate the independent source <u>AND</u> apply a test source of 1V at the terminals.

We then apply mesh analysis. Note the constraint of the dependent source v_x=-4i_2 (with the minus!)



$$-2v_x + 2(i_1 - i_2) = 0$$

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

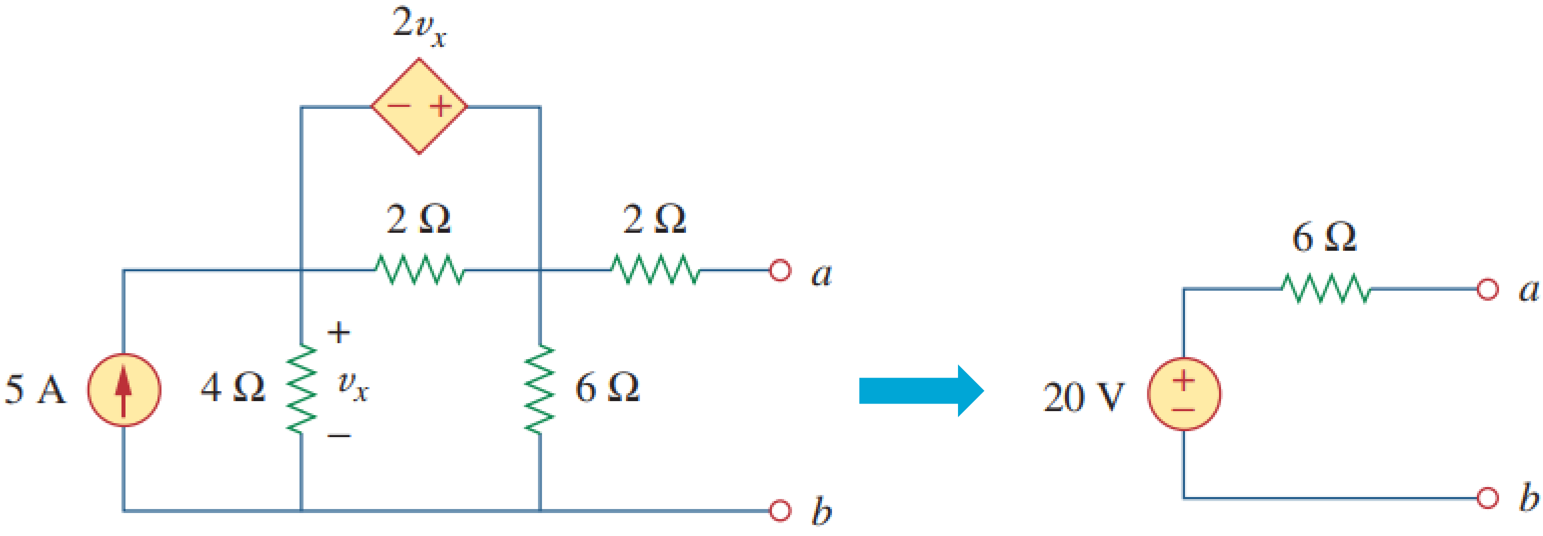
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

We want to find i_o as $R_{th}=v_o/i_o$, but $i_o=-i_3$

By solving the system, we find i_3=-1/6A
Hence R_{th}=**6Ω**

T&N: Dependent and Independent Sources together

We put together the two components to realize the Thevenin equivalent circuit as follows.



T&N: Dependent & Independent Sources together

To summarize the **general approach** to find R_{th}

1. Deactivate all independent sources;
2. Apply a test voltage v_T or test current i_T source to the designated terminals;
3. Calculate the terminal current i_T if a test voltage v_T source is used and vice versa;
4. Get the Thévenin resistance by $R_{th} = {}^{v_T}/_{i_T}$

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Maximum Power Transfer

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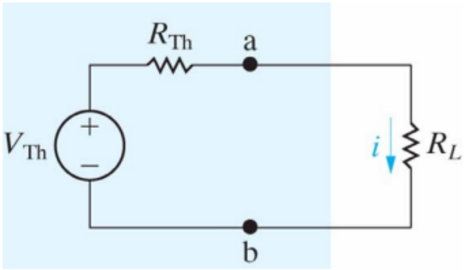
Maximum Power Transferred



- Consider a circuit (represented by a Thévenin equivalent) loaded with a resistance R_L. The power dissipation at R_L is indicated by *p*.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L,$$

- To find the value of R_L that leads to maximum power transfer, perform the derivative:

$$\frac{\boldsymbol{dp}}{\boldsymbol{dR_L}} = V_{th}^2 \frac{d}{dR_L}\left[R_L \frac{1}{(R_{th}+R_L)^2}\right] = \frac{1}{(R_{th}+R_L)^2} + R_L(-2)\left(\frac{1}{(R_{th}+R_L)^3}\right)$$

$$= \frac{(R_{th}+R_L) - 2R_L}{(R_{th}+R_L)^3} = \frac{\boldsymbol{R_{th} - R_L}}{\boldsymbol{(R_{th}+R_L)^3}}$$

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Maximum Power Transferred Derivation

- *P* is maximized when the derivative is equal to zero

  $\frac{R_{th}-R_L}{(R_{th}+R_L)^3}$ is equal to zero if the numerator is zero

  Hence $R_{th} = R_L$

- Therefore, the maximum power transfer is obtained when the load and the equivalent resistance are the same. Its value is:

$$p_{\max} = \left(\frac{V_{Th}}{R_{Th} + R_{Th}} \right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}}.$$

Summary

- Linearity & superposition
- Source transformations
- Thévenin and Norton theorems
  - replace a complex circuit by one source and one resistance
  - pay attention to the various types of sources occurring in the complex circuits!
- Maximum power transfer
  - occurs when $R_L = R_{Th}$, with R_{Th} being the equivalent resistance of the circuit to which the load R_L is connected

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Recap of Week 1-4

The toolbox is complete:

- Ohm's and Kirchhoff's laws
- Visual inspection
- Nodal analysis
- Mesh analysis
- Superposition
- Source Transformation
- Norton-Thévenin

So far, we applied these tools to *static* circuits containing only resistors.

Afterwards:

-*dynamic* circuits in time domain (2nd part of Lin Circ A)

-*dynamic* circuits in the phasor domain and Laplace domain (Lin Circ B)

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Next steps

- **SGH** (Self-Graded Homework assignments): posted today; submission due on Wednesday.
- **Seminar**: in groups on Tuesday & altogether on Friday.
- **1hour DRY-RUN EXAM** on Tuesday September 23rd afternoon

- **Next week**:
  - **MID-TERM EXAM** on Tuesday Sep 30th – Succes!

- **Week after next week**:

  -Circuits with op-amps

Thank you!

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