EE1C1 “Linear Circuits A”

Week 1.6

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Today

- Recapitulation of weeks 1.1–1.4
- Op-amps
  - definition, operation
  - ideal op-amps (nonideal op-amps)
  - typical circuits
- Summary and conclusions
- Next tasks

Join today's Vevox session

- Please open the session **158-351-038**
- You will have **30 seconds** for each question
- This session is fully anonymous
- Thank you for actively participating in the lecture!

Recap of weeks 1.1–1.4

- We have a full battery of tools:
  - Ohm’s law & Kirchhoff’s laws
  - Nodal/mesh analysis
  - Superposition
  - Source transformation
  - Thévenin/Norton theorems
  - Inspection (practice makes perfect)

Recap of weeks 1.1–1.4

- We know how to analyse:
  – circuits consisting of sources & resistances
- We do not know how to analyse:
  – circuits with other than proportional $i(t) \Leftrightarrow v(t)$ mappings (capacitances, inductances)
  – simple/complex transients, steady-state sinusoidal regime

Operational amplifiers (op-amps)

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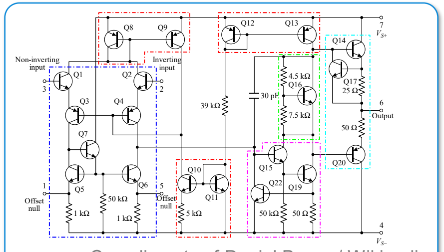
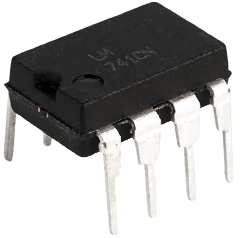
Passive and active electrical elements

- Passive elements (do not inject power)
  - resistances (resistors)
  - capacitances (capacitors)
  - inductances (inductors)
- Active elements (inject power)
  - (in)dependent sources (batteries, mains, etc.)
  - operational amplifiers ← Today!

Op-amps: basics



Compliments of Daniel Braun / Wikipedia



1968, David Fullagar

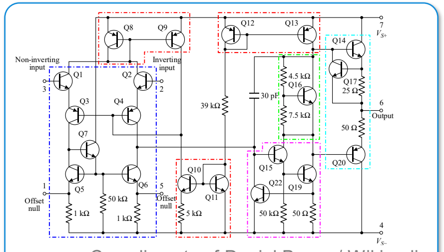
- Stands for “operational amplifier”

- Main purposes:
  - voltage-controlled voltage source (standard amplifiers)
  - in combination with other elements, it is at the core of other dependent sources
  - in combination with other elements, it can perform mathematical operations on analog signals (addition, subtraction, multiplication, differentiation, and integration)

Op-amps: basics

Main connexions:

1. The inverting input (indicated by a minus "–")
2. The noninverting input (indicated by a plus "+")
3. The output
4. The positive and negative power supplies



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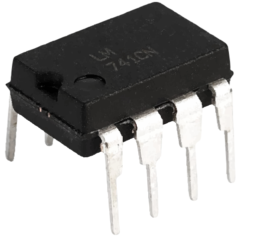
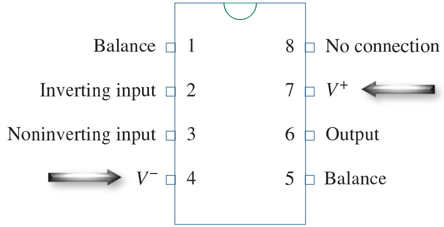
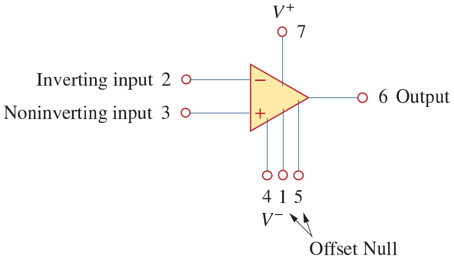
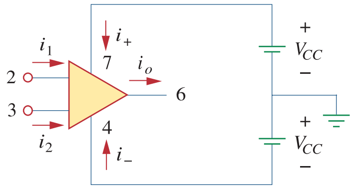

1968, David Fullagar





Op-amps: powering

- As an active element, the op-amp requires a power source



- In (our) circuit diagrams, the power supply terminals are often obscured (it is taken for granted that they must be connected)

- Most op-amps use two voltage sources, with a ground reference between them → this requires a positive and negative supply voltage

Op-amps: amplification (gain)

- The voltage output of an op-amp is proportional to the difference between the noninverting and inverting inputs:

$$v_0 = Av_\mathrm{d} = A\left(v_2 - v_1\right)$$

  - here, A is denoted as the open loop gain
  - ideally it is infinite
  - in real devices, it is finite, but very large: 10^5 to 10^8 volts/volt

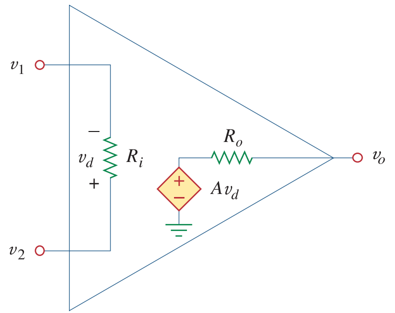
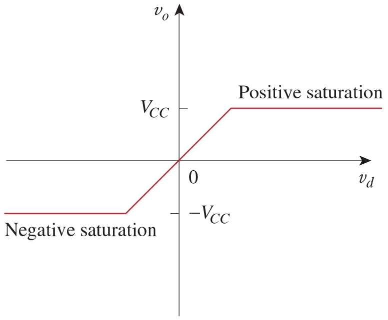


TABLE 5.1

Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

Op-amps: voltage saturation



- Ideally (for an ideal source), the output voltage would be unlimited
- In reality, the output cannot exceed the supply voltage

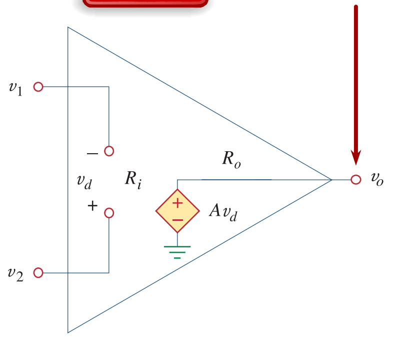
Recall:
$$v_0 = Av_{\mathrm{d}} = A\left(v_2 - v_1\right)$$

- When output should exceed the possible voltage range, the output remains at either the maximum or minimum supply voltage → saturation
- Outputs between these limiting voltages are referred to as the linear region

(Non)ideal op-amps: analysis

Op-amps: ideal op-amps

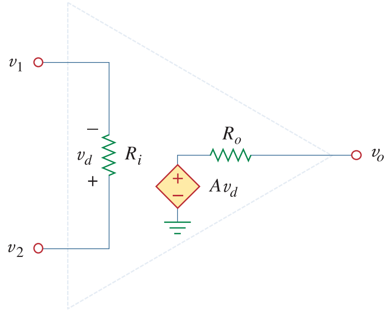
Output of voltage source!



- Parameters of the ideal op-amp:
  - an infinite open-loop gain A $(A \to \infty)$
  - input resistance between the inputs R_i is infinite $(R_\mathrm{i} \to \infty)$ ⟶ the op-amp absorbs no current and, thus, the voltage of any node attached to the input will not be affected
  - zero output resistance R_o
- Many modern op-amp come close to these ideal values:
  - very large gains, larger than 10^6
  - input resistances are often in the range $10^9\,\Omega$ to $10^{12}\,\Omega$

Op-amps: nonideal op-amps

- Nonideal op-amps?
- They must be accounted for via their equivalent circuit:
  - use the given input R_i, and output R_o resistances
  - use the given amplification A
  - examine the complete circuit with these parameters (quite likely, via nodal analysis)



Op-amps: feedback

- The op-amp voltage output: $v_0 = Av_\mathrm{d} = A\left(v_2 - v_1\right)$
- Ideal op-amp $A \to \infty$:
  - $v_2 - v_1 =$ finite $\longrightarrow$ $v_\mathrm{o} \to \infty$
  - $v_2 - v_1 \searrow 0$ $\longrightarrow$ $v_\mathrm{o} \searrow 0$, or $v_\mathrm{o} =$ finite, or $v_\mathrm{o} \to \infty$

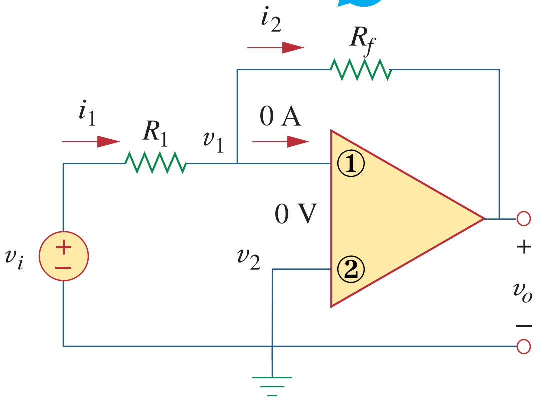
- The open-loop operation of an op-amp is unstable!
- Solution: feedback $\longrightarrow$ a part of the output is fed back
  - into the inverting input = negative feedback $\longrightarrow$ stable amplification
  - into the noninverting input = positive feedback $\longrightarrow$ stable oscillation

not handled in this course

Op-amps: feedback

Consider the circuit at the right.

- By knowing that the circuit makes use of negative feedback, indicate which of the two terminals is the positive (noninverting) input?

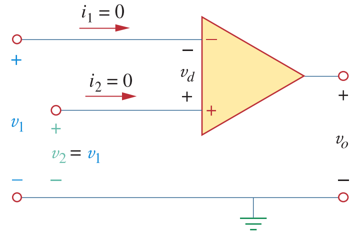


a) Input 1

b) Input 2

c) I cannot indicate

Op-amps: zero conditions



- If the op-amp is ideal, the current into the input terminals is zero: $i_1 = 0\,,\ i_2 = 0$
- If, and only if, there is also feedback, the voltage across the input terminals is equal to zero: $v_1 = v_2$
- Textbook: Two important ~~characteristics of~~ properties applicable to the ideal op-amp are:

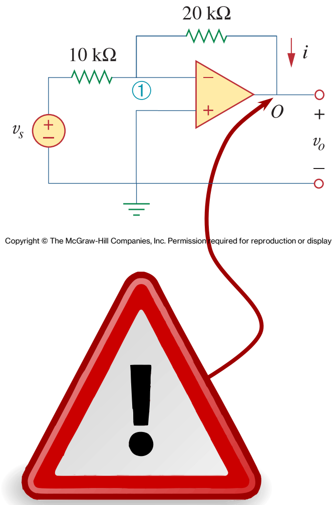
$$i_1 = 0,\ \ i_2 = 0$$
$$v_1 = v_2$$

zero conditions

"golden" conditions

Let's check!

Consider the circuit at the right, in which the op-amp is an ideal one.

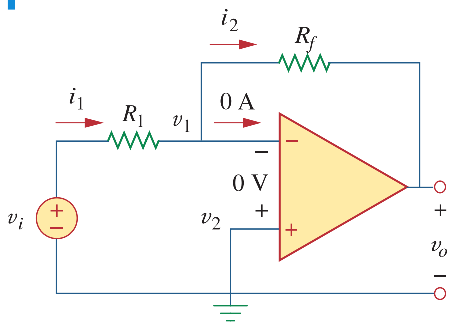


Never @ output!

- By knowing that $v_s = 2\text{V}$, determine:
  - the output voltage v_o
  - the current i
- *Hint:* apply the zero conditions at the node ①
- Solution: $v_o = \text{-}4\text{V}$ and $i = 0.2\text{mA}$

Typical op-amp circuits

Typical circuits: inverting amplifier



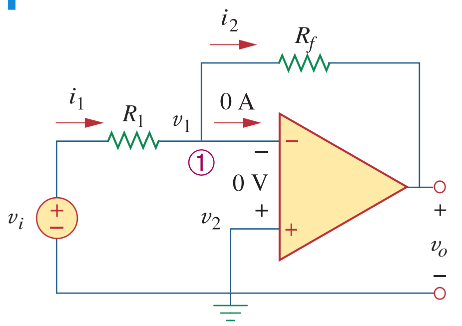
- Topology:
  - the noninverting input is grounded
  - the inverting input is connected via a feedback resistance R_f to the output
  - the input voltage v_i is applied via a resistance R_1 to the inverting input

- The op-amp is taken to be ideal

Typical circuits: inverting amplifier

- Analysis:

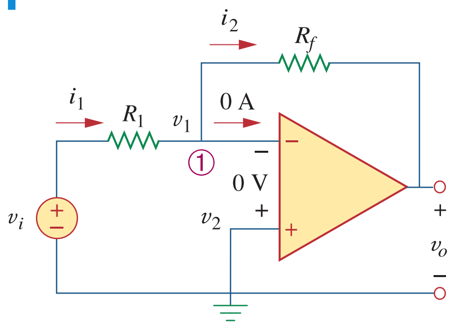
1. Apply KCL at node ①: $i_1 = i_2$
2. The voltage across the input terminals is zero: $v_1 = v_2 = 0$



3. Substituting yields: $$\frac{v_\mathrm{i} - v_1}{R_1} = \frac{v_1 - v_\mathrm{o}}{R_\mathrm{f}}$$

- Final result: $$\frac{v_\mathrm{i}}{R_1} = -\frac{v_\mathrm{o}}{R_\mathrm{f}} \rightarrow v_\mathrm{o} = -\frac{R_\mathrm{f}}{R_1} v_\mathrm{i}$$

Typical circuits: inverting amplifier



Conclusions:

1. The gain is the ratio of the feedback resistances R_f and R_1
   - if $R_\mathrm{f} = 0$ ⟶ the output is zero (v_o is grounded) and the output current?
   - setting $R_1 = 0$ is not allowed (this would ground the input voltage source)

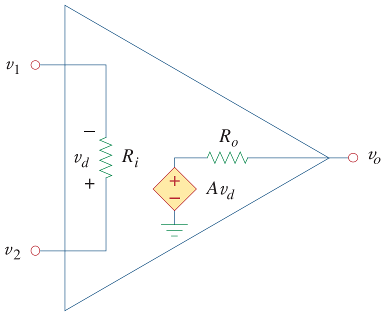
$$v_\mathrm{o} = -\frac{R_\mathrm{f}}{R_1} v_\mathrm{i}$$

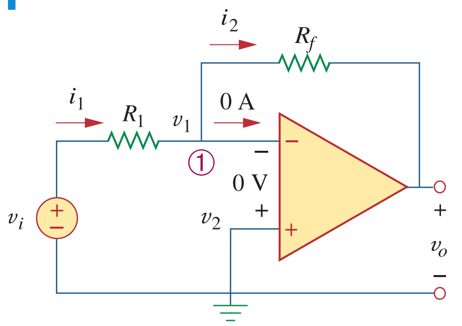
2. The polarity of the output is the reverse of the input, thus the name “inverting” amplifier

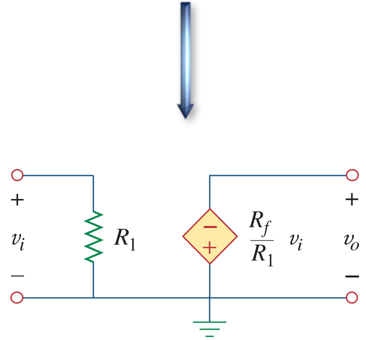
Typical circuits: inverting amplifier

Equivalent circuit of the inverting amplifier

- Recall that

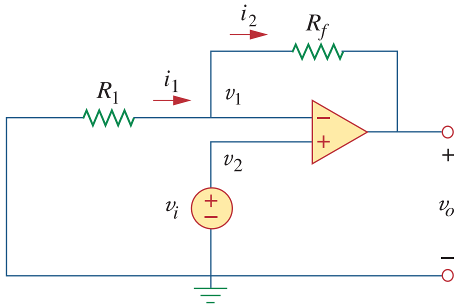






- Important observation: the amplifier's input resistance is finite!

Typical circuits: noninverting amplifier

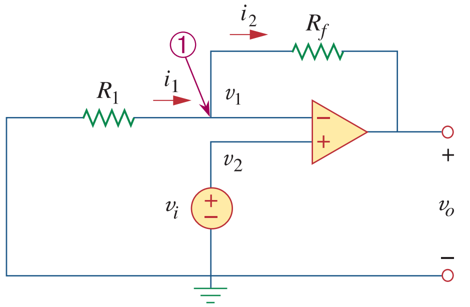


Topology:

- the input voltage v_i is connected to the noninverting input
- the inverting input is connected via a feedback resistance R_f to the output and via a resistance R_1 to the ground

- The op-amp is taken to be ideal

Typical circuits: noninverting amplifier



Analysis:

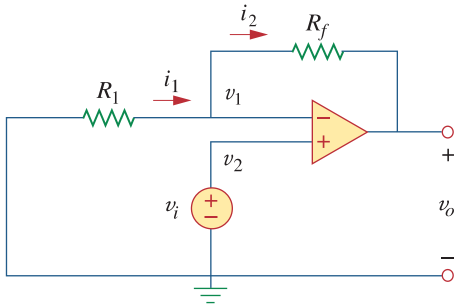
1. Apply KCL at node ①: $i_1 = i_2$

$$v_1 = \frac{R_1}{R_1 + R_\mathrm{f}} v_\mathrm{o}$$

2. The voltage across the input terminals is zero: $v_1 = v_2 = v_\mathrm{i}$

- Final result: $v_\mathrm{o} = \frac{R_1 + R_\mathrm{f}}{R_1} v_\mathrm{i}$ $= \left(1 + \frac{R_\mathrm{f}}{R_1}\right) v_\mathrm{i}$

Typical circuits: noninverting amplifier



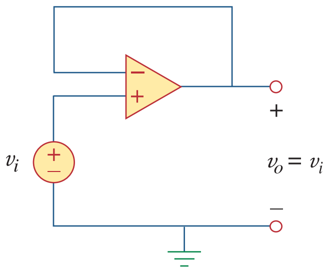
Conclusions:

1. The gain is the ratio of the feedback resistances R_f and R_1 + 1!!!
   - if $R_\mathrm{f} = 0$ the output is directly connected to the inverting input (next slide)
   - setting $R_1 = 0$ is not allowed (this would again ground the input voltage source)
2. The minimum gain is 1
3. The polarity of the output is the copy of the input, thus the name "noninverting" amplifier

$$v_\mathrm{o} = \left(1 + \frac{R_\mathrm{f}}{R_1}\right) v_\mathrm{i}$$

Typical circuits: noninverting amplifier

- By setting the feedback resistance $R_\mathrm{f} = 0$, the gain becomes 1 (the resistance R_1 is redundant and was eliminated)
- The new configuration is referred to as voltage follower (unity-gain amplifier)
- It copies the input signal by providing high input impedance → expedient for impedance matching or for measuring weak source signals



Typical circuits: analysis

1. For ideal op-amps:

- Do we have negative feedback? yes → go on | no → stop
- Apply nodal/mesh analysis to the circuit
- Account for the zero conditions at the input terminals:
  - the current into the input terminals is zero
  - the voltage across the input terminals is zero

  } fill them in the equations
- Solve the resulting equations

2. For nonideal op-amps: replace the op-amp by its equivalent circuit and solve the complete circuit

Typical circuits: analysis

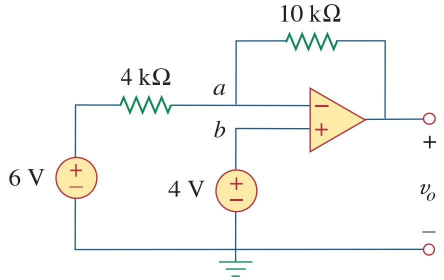
1. For ideal op-amps:

- Do we have negative feedback? yes → go on | no → stop
- Apply nodal/mesh analysis to the circuit
- Account for the zero conditions at the input terminals:
  - the current into the input terminals is zero
  - the voltage across the input terminals is zero

  } fill them in the equations
- Solve the resulting equations

Never apply nodal analysis at the output terminal if $R_o = 0$!
(the output of a dependent voltage source)

Typical circuits: analysis

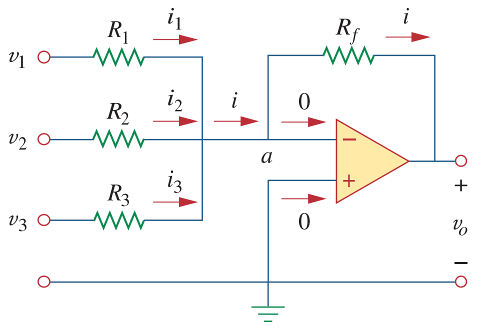


- Consider the circuit at the right, in which the op-amp is an ideal one.
- Determine the output voltage v_o.

1. Is there negative feedback? ✓
2. Apply the zero conditions
   - input current $i_a = 0$
   - voltage across the input terminals is zero: $v_a = v_b = 4\,\mathrm{V}$
3. Apply KCL @ a: $\frac{v_a - 6}{4k} + \frac{v_a - v_o}{10k} = 0 \longrightarrow 7v_a = 30 + v_o$

$v_o = -1\,\mathrm{V}$

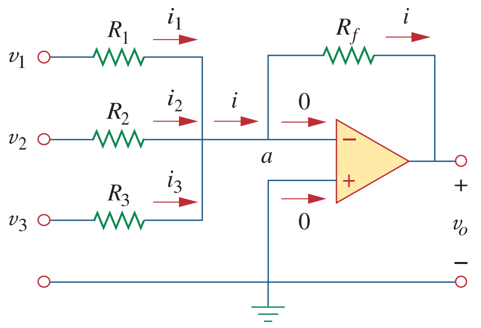
Typical circuits: summing amplifier



- Op-amps can combine amplification and addition
- Topology: inverting amplifier **+** combination of inputs with own resistances
- Analysis:

1. Ohm's law at the inputs: $i_1 = \frac{v_1 - v_a}{R_1}$, $i_2 = \frac{v_2 - v_a}{R_2}$, $i_3 = \frac{v_3 - v_a}{R_3}$
2. KCL @ a: $i_1 + i_2 + i_3 = i$

Typical circuits: summing amplifier



- Analysis (continued):

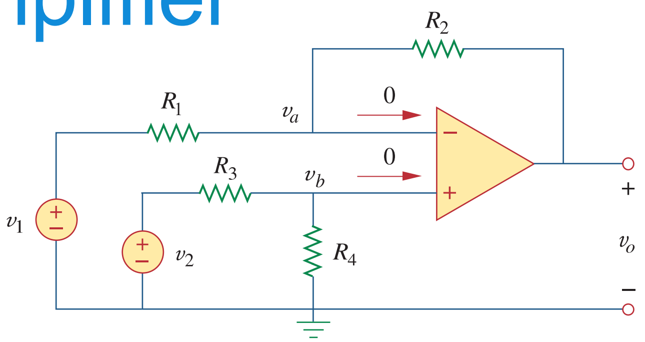
3. Account for the zero conditions
   - voltage between terminals is zero → $v_\mathrm{a} = 0$
   - current in the terminal is zero

$$i = \frac{v_\mathrm{a} - v_\mathrm{o}}{R_\mathrm{f}} = i_1 + i_2 + i_3$$

4. A bit of algebra: $v_\mathrm{o} = -\left(\frac{R_\mathrm{f}}{R_1} v_1 + \frac{R_\mathrm{f}}{R_2} v_2 + \frac{R_\mathrm{f}}{R_3} v_3\right)$

- Conclusion: not only a sum, but a weighted sum!

Typical circuits: difference amplifier



- Subtraction: based on the output being proportional to the difference between inputs
- Topology: combination of inverting + noninverting amplifier
- Analysis (zero input currents implied)

1. KCL @ a: $\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$ → $v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$
2. Voltage divider @ b: $v_b = \frac{R_4}{R_3 + R_4} v_2$

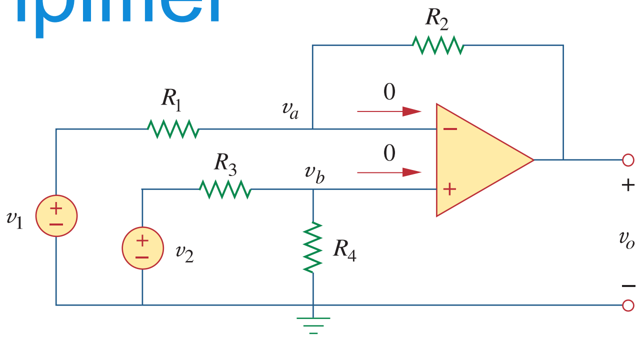
Typical circuits: difference amplifier

3. Using $v_a = v_b$ + a bit of algebra

$$v_o = \frac{R_2\left(1 + R_1/R_2\right)}{R_1\left(1 + R_3/R_4\right)} v_2 - \frac{R_2}{R_1} v_1$$

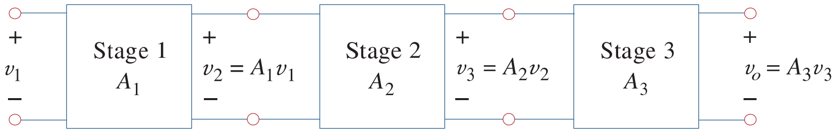
4. Select $R_1 = R_2$ and $R_3 = R_4$ it yields a subtractor

$$v_o = v_2 - v_1$$



Typical circuits: cascaded op-amps

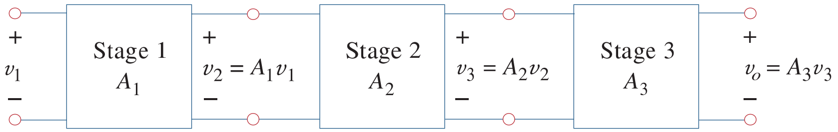
- It is common to use multiple op-amp circuits chained one after the other together



- This head-to-tail connection is referred to as cascading
- Each amplifier is referred to as a stage

- Purposes:
  – higher gain under stable amplification
  – mixing high gain and input resistance matching

Typical circuits: cascaded op-amps



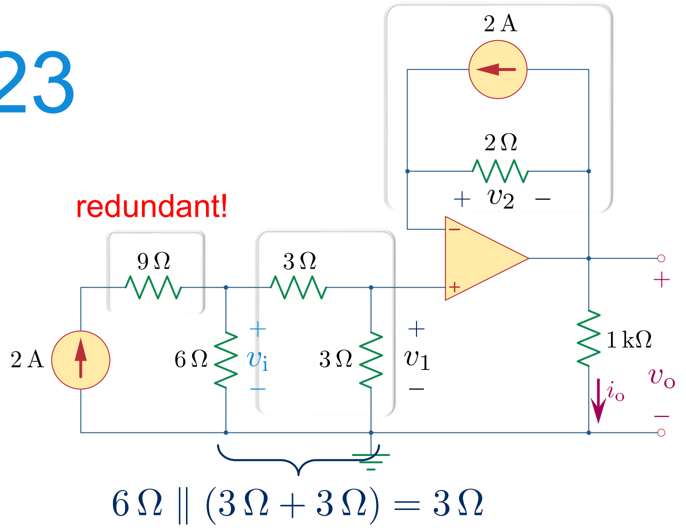
- It is common to use multiple op-amp circuits chained one after the other together
- This head-to-tail connection is referred to as cascading
- Each amplifier is referred to as a stage

- Total gain: $A_{\text{tot}} = A_1 A_2 A_3 (\ldots)$

Exam(ple) – from 2022-2023

Consider the circuit at the right.

- Calculate the current i_o in the circuit, by assuming the op-amp to be ideal



- Circuit analysis: (everything we know about the problem)
  - is the op-amp ideal? ✓
  - is there feedback? ✓
  - what are the voltages applying to the op-amp? ✓

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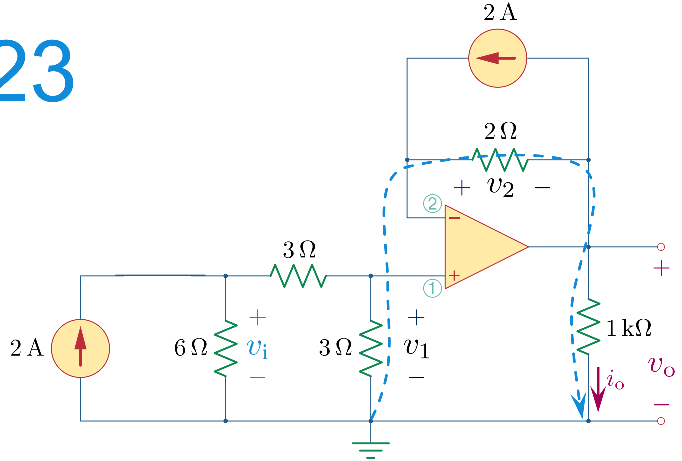
Exam(ple) – from 2022-2023

- Applicable voltages:
  - feedback: $v_2 = 4\text{V}$
  - input: $v_\text{i} = 6\text{V}$
  - noninverting input: $v_1 = 3\text{V}$

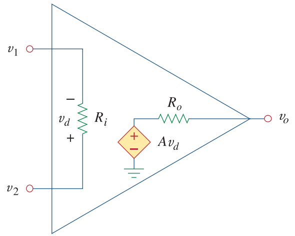


- Voltage across the input terminals is zero → between ① and ②

→ KVL on the loop → $v_\text{o} = v_1 - v_2 = -1\,\text{V}$

- Ohm's law: $i_\text{o} = -\dfrac{1}{1\text{k}} = -1\,\text{mA}$

Summary of the day



- Ideal op-amps:
  - amplification $A \to \infty$
  - resistances: input $R_\mathrm{i} \to \infty$, output $R_\mathrm{o} \searrow 0$
- Zero conditions:
  - input currents are zero
  - with a (negative) feedback, the voltage across the input terminals is zero $v_2 - v_1 \searrow 0$

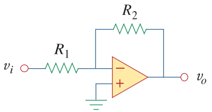
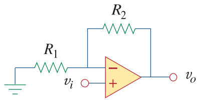
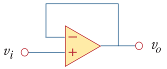
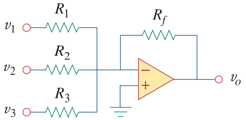
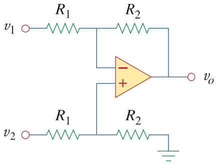
Summary of the day

- Typical op-amp circuits

Hint: Do not recall formulas, recall how to derive them!

TABLE 5.3

Summary of basic op amp circuits.

Op amp circuit	Name/output-input relationship
	Inverting amplifier $v_o = -\frac{R_2}{R_1} v_i$
	Noninverting amplifier $v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$
	Voltage follower $v_o = v_i$
	Summer $v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$
	Difference amplifier $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

Next tasks

- Please do the SGH4
- Seminars of Tuesday (only 3 rooms!) and Friday (new location – Flux / Hall A!)
- Next week: new electrical elements: capacitances and inductances

Thank you!