EE1C1 “Linear Circuits A”

Week 1.8

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TU Delft

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Mathematics and Computer Science

Today

- Recap: Capacitance & capacitor; inductance & inductor; C–C and L–L interconnections
- New topics:
  - First-order transient circuits: RL and RC
  - Step response
  - Exam exercise example
- Summary and conclusions
- Next tasks
- Your opinion counts!

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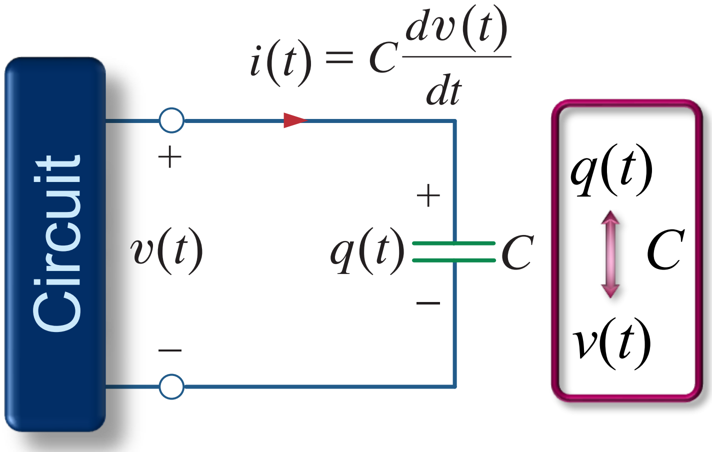
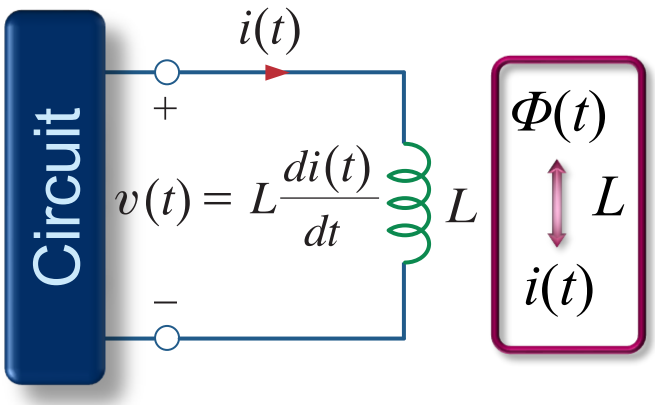
Recap of week 1.7

Capacitance / capacitors



Inductance / inductors



Transient circuits

First-order circuits – RC and RL

Transient analysis

- Thus far we only examined the ‘steady state’
  - currents and voltages are constant

- Transient analysis amounts to examining circuits when a ‘disturbance’ is induced

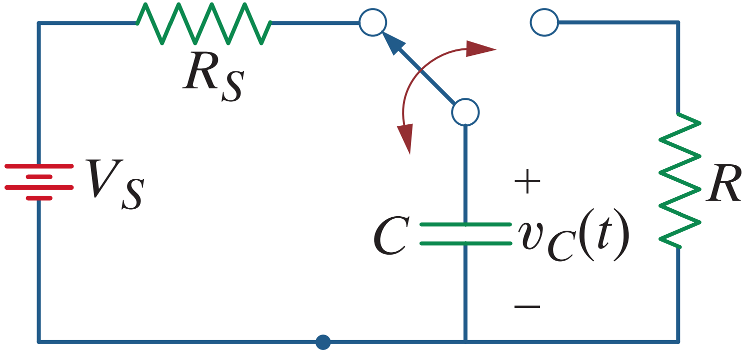
- It is carried out in time-domain

Transient analysis

- Capacitances and inductances can store and release energy
  - abrupt changes cannot have an instantaneous effect
  - the process depends on the rest of the circuit
- Time constant:
  - indicates how fast disturbances propagate through a circuit
  - light switch  ⟷ thermostat 
- First-order circuits:
  - a combination of one resistance and one capacitance ⟶ RC
  - a combination of one resistance and one inductance ⟶ RL

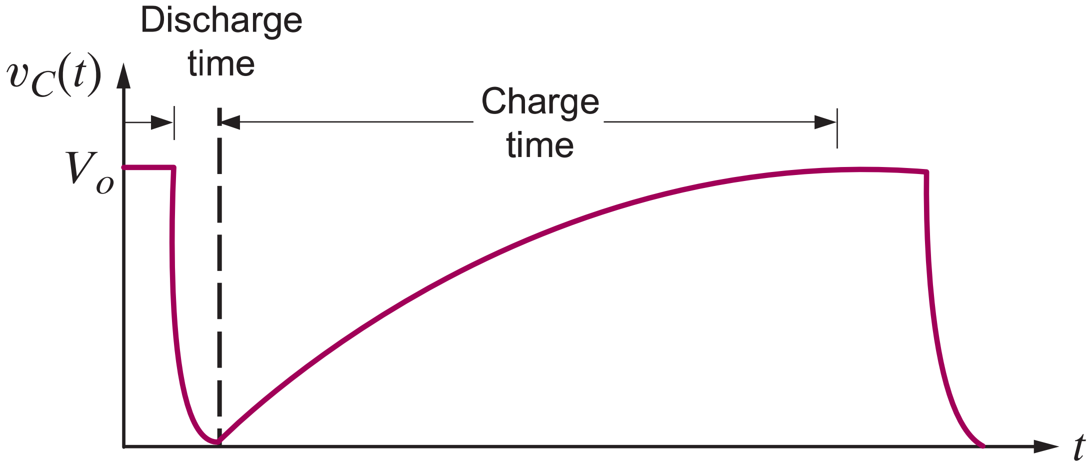
Transient analysis

- Camera with a flash

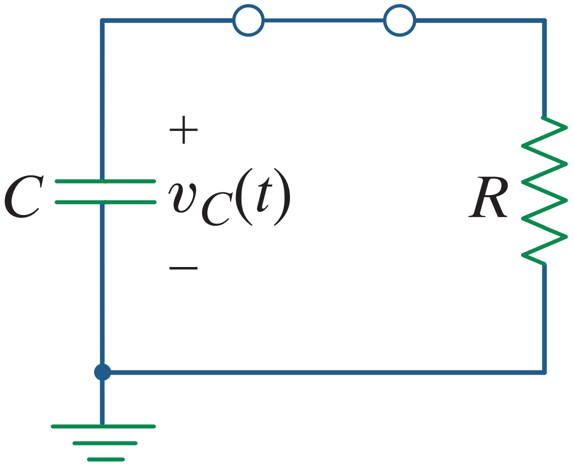


[(R) Xenon lamp]

- Process:
  - charging
  - flash!
  - charging



Transient analysis



- The discharge part:
  - can be construed as a capacitance that releases its energy in a resistance R
- KCL:

$$C\frac{dv_C(t)}{dt}+\frac{v_C(t)}{R}=0 \quad \longrightarrow \quad \frac{dv_C(t)}{dt}+\frac{1}{RC}v_C(t)=0$$

- The solution:

$$v_C(t)=V_0e^{-t/RC}$$

Differential equations:
a math-in-a-nutshell moment

Differential equations

- Analysing such circuits amounts to solving a differential equation

$$\frac{dv_C(t)}{dt} + \frac{1}{RC} v_C(t) = 0$$

- General form:

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

- Task: find $x(t)$

Differential equations

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

Linear, differential equations with constant coefficients ⟶ it holds:

- When one knows a (particular) solution for the general form

$$x(t) = x_{\mathrm{p}}(t) \quad \mathrm{p} \rightarrow \text{"particular"}$$

$$\frac{dx_p(t)}{dt} + ax_p(t) = f(t)$$

- AND the solution of the homogeneous form

$$x(t) = x_{\mathrm{h}}(t) \quad \mathrm{h} \rightarrow \text{"homogeneous"}$$

$$\frac{dx_h(t)}{dt} + ax_h(t) = 0$$

- THEN the complete solution of the differential equation is:

$$x(t) = x_p(t) + x_h(t)$$

Differential equations

- For the time being, we only consider constant right-hand side terms of the form $f(t) = A$
- General form: $$\frac{dx_p(t)}{dt} + ax_p(t) = A$$
- Homogeneous form: $$\frac{dx_h(t)}{dt} + ax_h(t) = 0$$

Differential equations

- Since the right-hand side term is constant, we may assume that the particular solution $x_{\mathrm{p}}(t)$ is also constant

$$\frac{dx_p(t)}{dt} + ax_p(t) = A$$

$$x_p(t) = K_1 \longrightarrow K_1 = \frac{A}{a}$$

Differential equations

- From the homogeneous equation it follows that:

$$\frac{dx_h(t)}{dt} + ax_h(t) = 0 \longrightarrow \frac{dx_h(t)/dt}{x_h(t)} = -a \longrightarrow \frac{d}{dt}\left[\ln x_h(t)\right] = -a$$

c collects constants from both sides

- Consequently:

$$\ln x_h(t) = -at + c \longrightarrow x_h(t) = K_2 e^{-at}$$

$K_2 = e^{c}$

Differential equations

- Combining the two solutions: $x_p(t) = K_1 = \dfrac{A}{a}$ **&** $x_h(t) = K_2 e^{-at}$

$$x(t) = x_p(t) + x_h(t) = \frac{A}{a} + K_2 e^{-at}$$

- K_2 can be determined if we knew $x(t)$ at a given moment
- Problem solved!

Differential equations

- In general, we can express the solution of our first-order differential equation as

$$x(t) = K_1 + K_2 e^{-t/\tau}$$

- K_1 is the "steady-state" or the stationary solution for $t \to \infty$

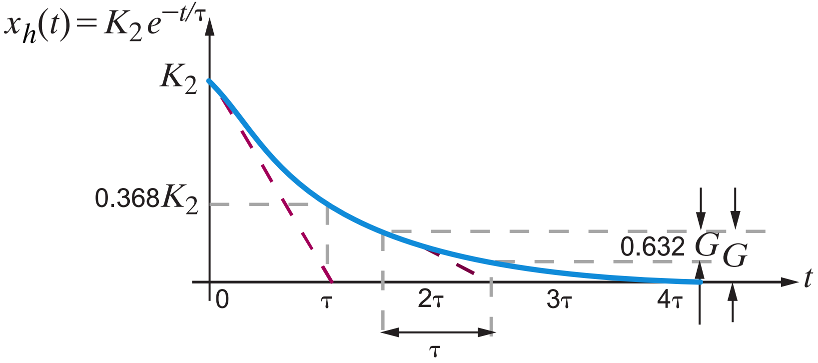
- τ is the circuit's time constant ⟶ it measures how fast the second term vanishes

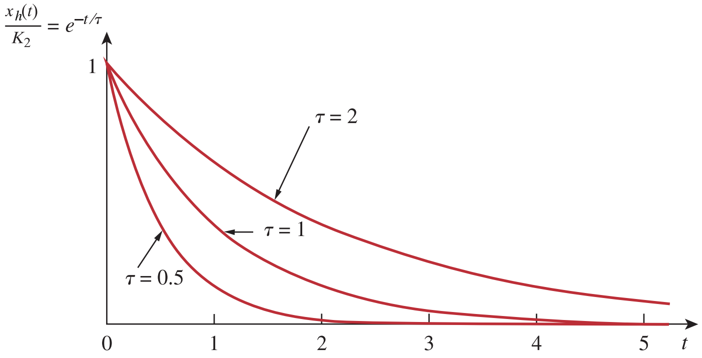
Differential equations: time constant

- After each τ, the quantity $x_\mathrm{h}(t)$ drops by e (Euler's number)

Values of $x_h(t)/K_2 = e^{-t/\tau}$

t	$x_h(t)/K_2$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674





- The smaller the time constant τ, the more abrupt the transition

Method 1: Differential equations

+ examples

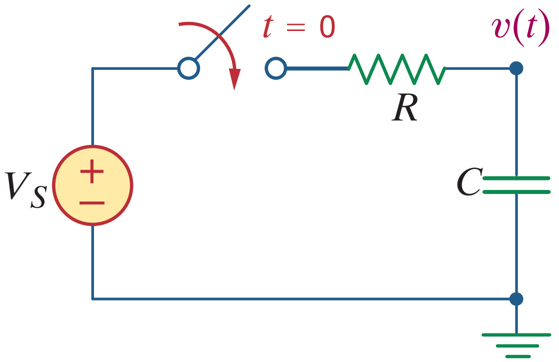
Algorithm

- Write the KCL/KVL equations pertaining to
  – the voltage over the capacitance
  – the current through the inductance
- Fill in the generic solution:

$$x(t) = K_1 + K_2 e^{-t/\tau}$$

- Identify the coefficients and make use of the initial values

Example 1: loading of a capacitance



- KCL: $$C\frac{dv(t)}{dt}+\frac{v(t)-V_S}{R}=0$$

$$\frac{dv(t)}{dt}+\frac{v(t)}{RC}=\frac{V_S}{RC}$$

- Fill in the generic solution: $$x(t)=K_1+K_2e^{-t/\tau}$$

$$-\frac{K_2}{\tau}e^{-t/\tau}+\frac{K_1}{RC}+\frac{K_2}{RC}e^{-t/\tau}=\frac{V_S}{RC}$$

Example 1: loading of a capacitance

$$-\frac{K_2}{\tau}e^{-t/\tau}+\frac{K_1}{RC}+\frac{K_2}{RC}e^{-t/\tau}=\frac{V_S}{RC}$$

- The constant terms and the coefficients in the exponentials are:

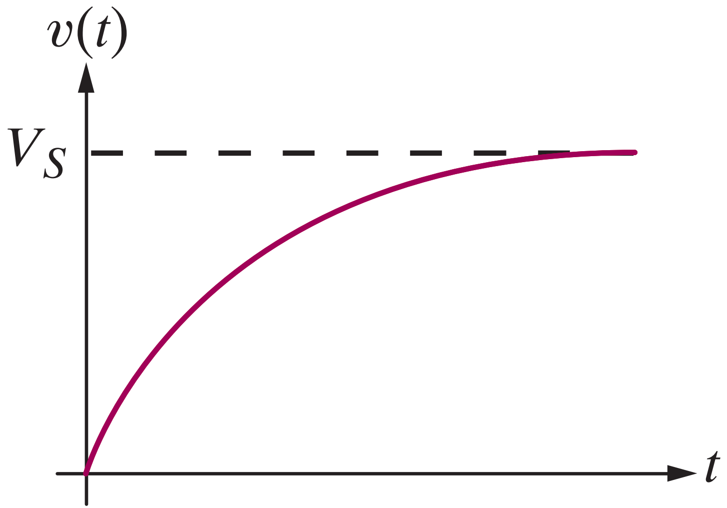
  $K_1 = V_S$ and $\tau = RC$

- Thus: $v(t) = V_S + K_2 e^{-\frac{t}{RC}}$
- K_2 follows by observing the initial condition: the voltage over the capacitance is zero at $t = 0$ → $K_2 = -V_S$
- We can now conclude that: $v(t) = V_S - V_S e^{-t/RC} = V_S(1 - e^{-t/RC})$

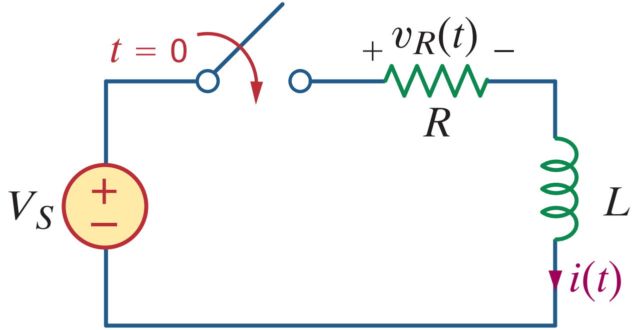
Example 1: loading of a capacitance

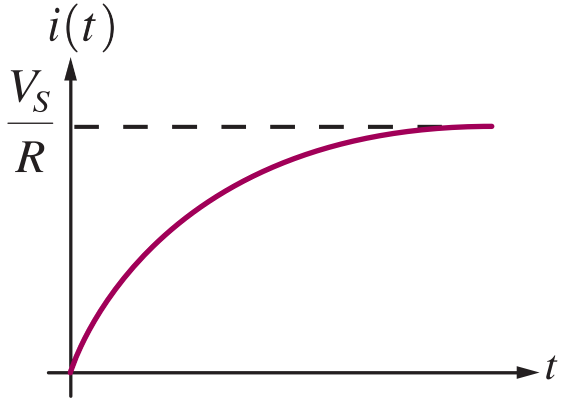
- Plot of the solution

$$v(t) = V_S(1 - e^{-t/RC})$$



RL circuit: practically the same

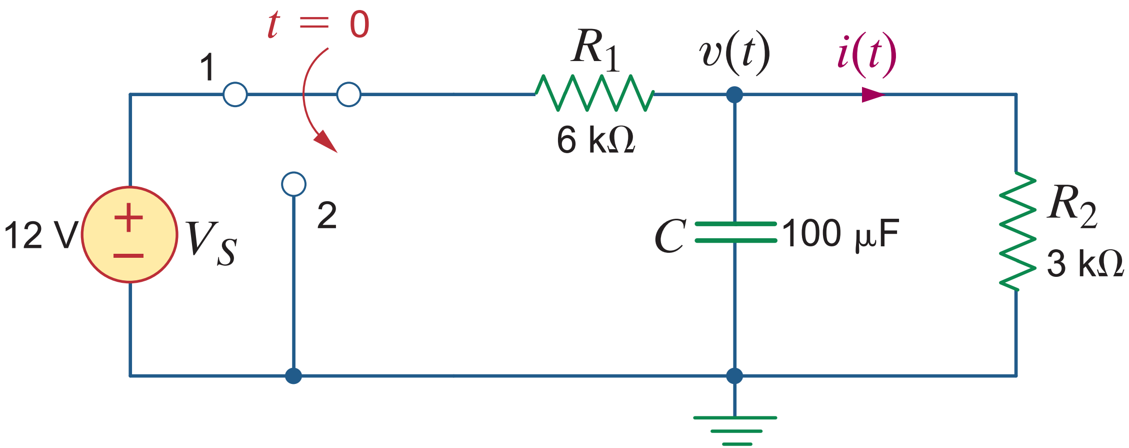




$$i(t) = \frac{V_S}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Example 2: discharge of a capacitance

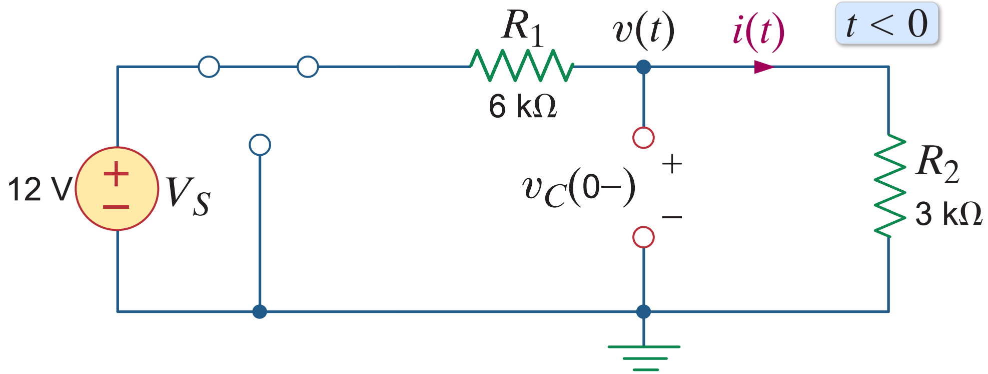
- The switch is taken to have been for very long in position 1



- Determine the current $i(t)$ for $t > 0$

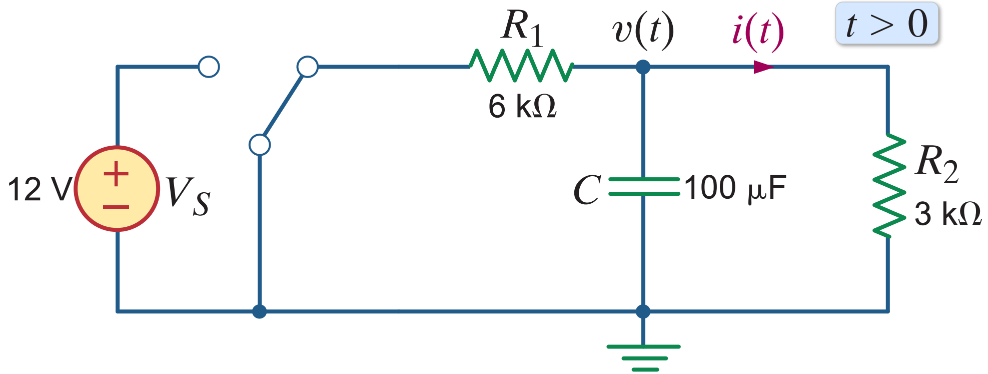
Example 2: discharge of a capacitance

- For $t \uparrow 0$ we have:



- The capacitance is fully charged → the current is zero
- The initial voltage is: $v(0-) = 12\left(\frac{3k}{6k+3k}\right) = 4\,\text{V}$

Example 2: discharge of a capacitance



- For $t > 0$ we have:

- KCL: $$\frac{v(t)}{R_1} + C\frac{dv(t)}{dt} + \frac{v(t)}{R_2} = 0$$

- With the given values: $$\frac{dv(t)}{dt} + 5v(t) = 0$$

- $R_1 = 6\ \text{k}\Omega$
- $R_2 = 3\ \text{k}\Omega$
- $C = 100\ \mu\text{F}$

Example 2: discharge of a capacitance

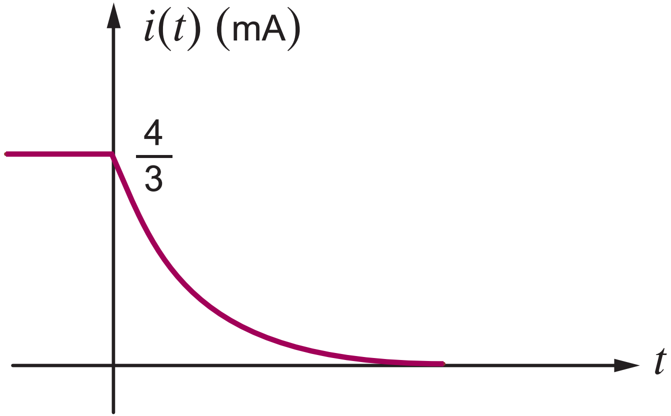
- This is a homogeneous equation: $\frac{dv(t)}{dt} + 5v(t) = 0$

- The solution reads: $v(t) = K_2 e^{-t/\tau}$

- Filling in the values yields: $v(t) = K_2 e^{-t/0.2}\ \text{V}$

Example 2: discharge of a capacitance

- Filling in the initial voltage $v(0^-) = 4\text{V}$ → $v(t) = 4e^{-t/0.2}\ \text{V}$
- Thus, the current is: $i(t) = \dfrac{v(t)}{R_2}$ → $i(t) = \dfrac{4}{3}e^{-t/0.2}\ \text{mA}$
- Plot:



Method 2: Algorithmic

+ example

Algorithm

1) Assume that the solution has the form: $x(t) = K_1 + K_2 e^{-t/\tau}$

2) Assume that the circuit is in 'steady state' before the change occurs → express the current through L or the voltage across C just before the switch switches

3) Make use of the fact that the current or the voltage cannot be discontinuous: $v_C(0+) = v_C(0-)$

$$i_L(0+) = i_L(0-)$$

Algorithm

4) Redraw the circuit in the new 'steady state' that holds for $t \to \infty$ → practically, this applies for $t > 5\tau$ → solve the new circuit → $x(t)\big|_{t>5\tau} = x(\infty)$

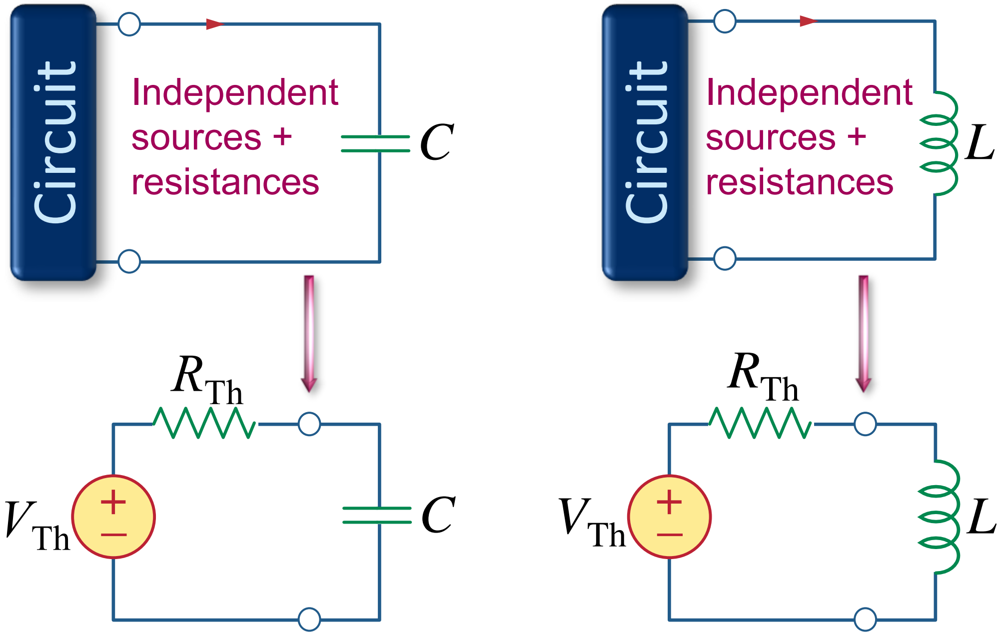
5) The time constant holds for all currents and voltages ❓

It is then easier to reduce the circuit to a voltage source, a resistance and a reactive element (capacitance or inductance) → the Thévenin equivalent at the reactive element's terminals: $\tau = R_{Th}C$ and $\tau = L/R_{Th}$

Algorithm

- Assume that "Circuit" contains only independent sources & resistances

- It can be replaced by its Thévenin equivalent



- The canonical RC or RL circuits! → **standard solution**

Algorithm

6) The constants at step 1 are now determined via the results from the steps 3, 4 and 5 $x(0+) = K_1 + K_2$ and $x(\infty) = K_1$

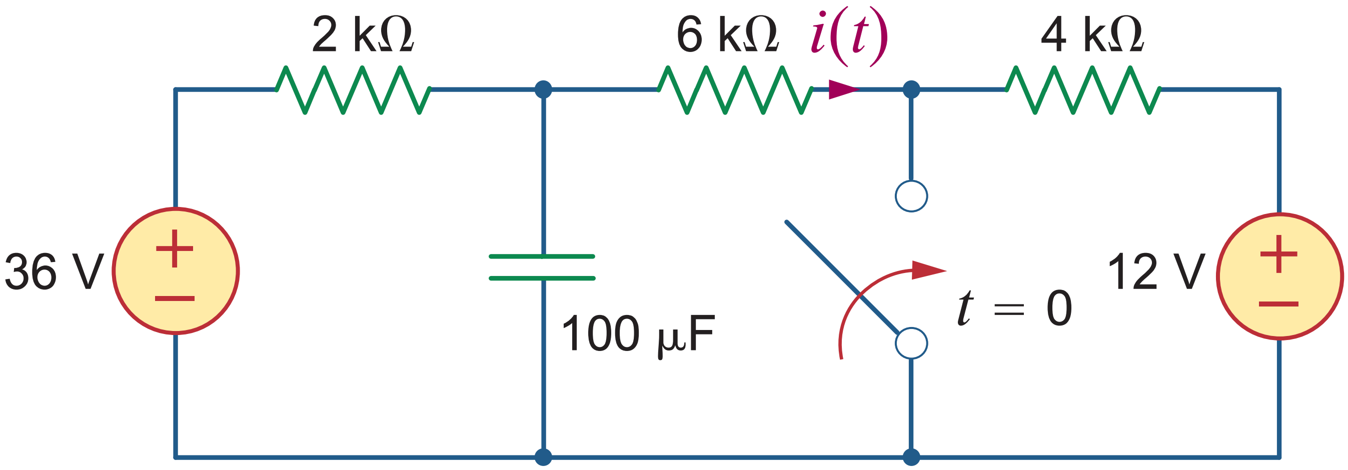
This entails $K_1 = x(\infty)$ and $K_2 = x(0+) - x(\infty)$

The final solution is: $x(t) = x(\infty) + \left[x(0+) - x(\infty)\right]e^{-t/\tau}$

Initial – final values formula

Example 3: complex circuit

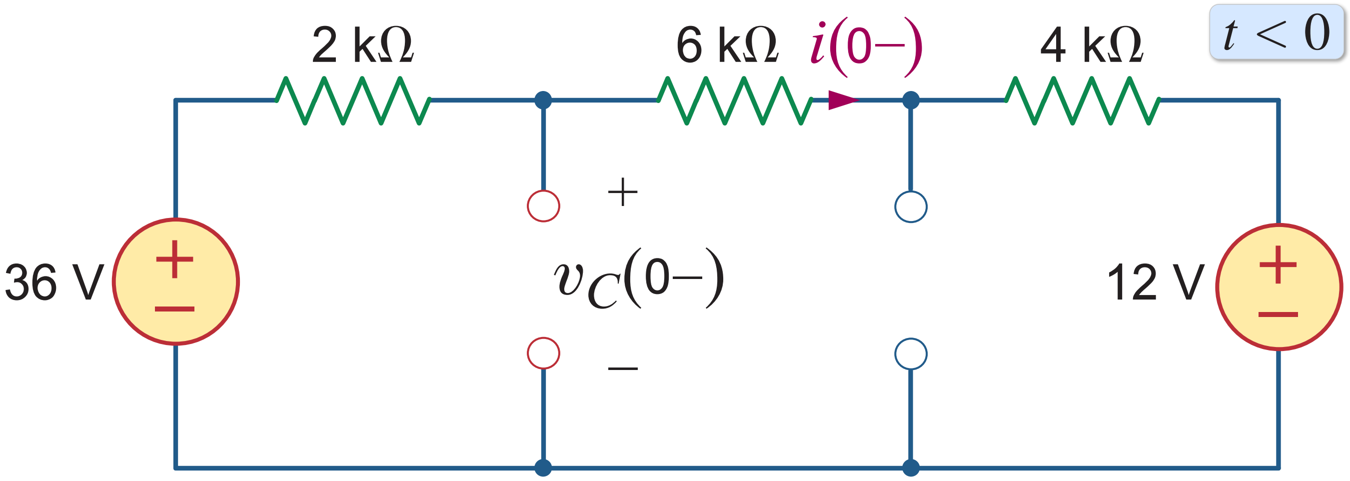
- Determine the current $i(t)$ for $t > 0$



- STEP 1: $i(t)$ is of the form: $i(t) = K_1 + K_2 e^{-t/\tau}$

Example 3: complex circuit

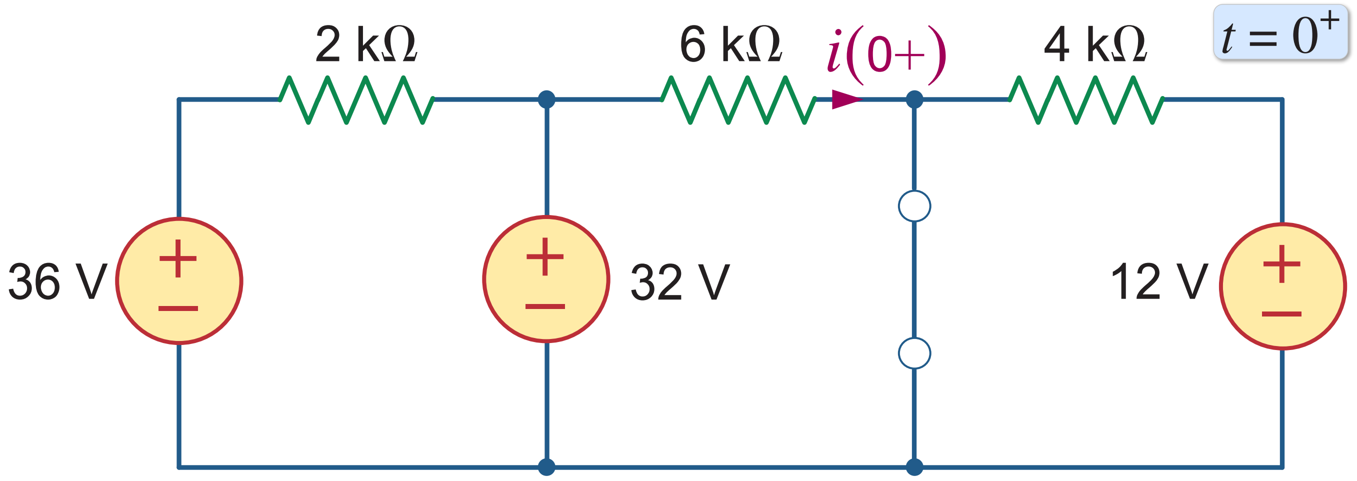
- STEP 2: Draw the circuit just before the switching moment



$$v_C(0\text{-}) = 36 - (2\text{k})(2\text{m}) = 32 \text{ V}$$

Example 3: complex circuit

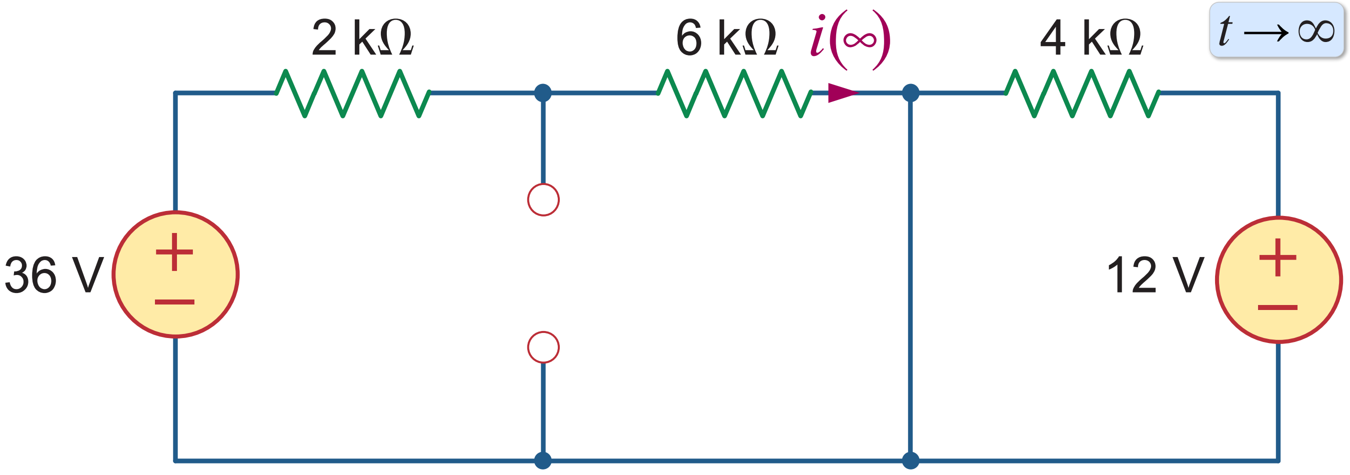
- STEP 2: Draw the circuit just after the switching moment



$$i(0+) = \frac{32}{6k} = \frac{16}{3} \text{ mA}$$

Example 3: complex circuit

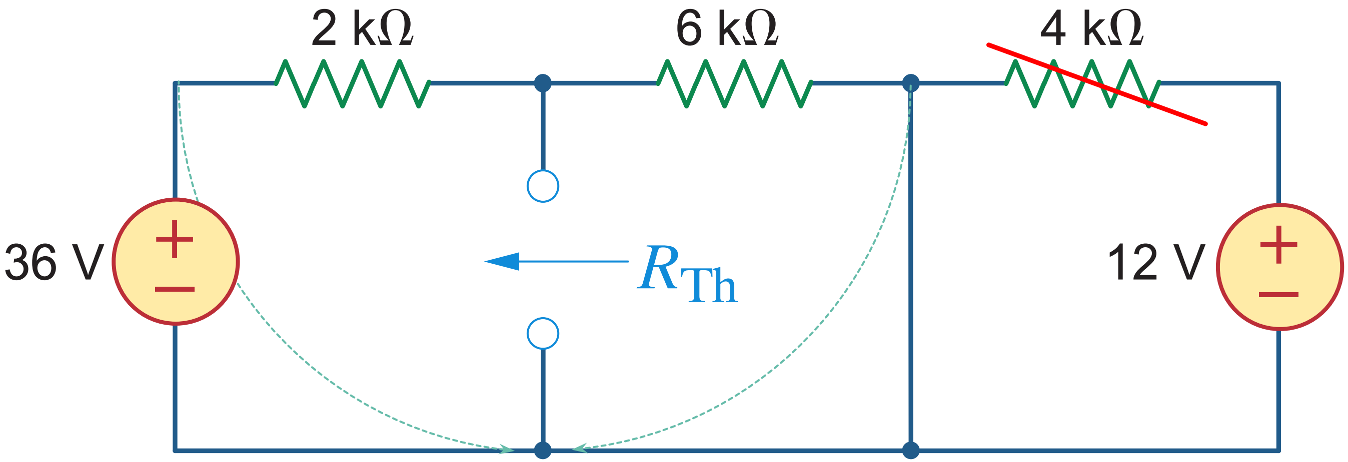
- STEP 4: The circuit in 'steady state', valid for $t > 5\tau$



$$i(\infty) = \frac{36}{2k + 6k} = \frac{9}{2} \text{ mA}$$

Example 3: complex circuit

- STEP 5: Determine R_{Th} at the capacitor's terminals



$$R_{Th} = \frac{(2k)(6k)}{2k+6k} = \frac{3}{2}\ \text{k}\Omega \longrightarrow \tau = R_{Th}C = \left(\frac{3}{2}\right)(10^3)(100)(10^{-6}) = 0.15\ \text{s}$$

Example 3: complex circuit

- STEP 6: Make use of the steps 3, 4 and 5

$$K_1 = i(\infty) = \frac{9}{2}\ \text{mA}$$

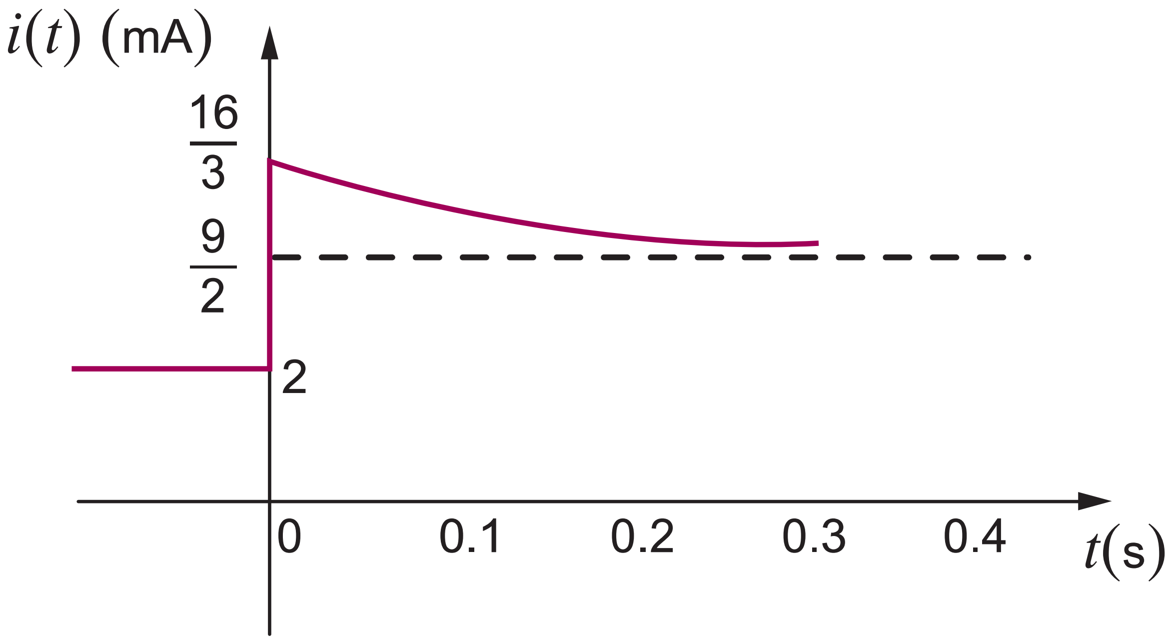
$$K_2 = i(0+) - i(\infty) = i(0+) - K_1 = \frac{16}{3} - \frac{9}{2} = \frac{5}{6}\ \text{mA}$$

- Hence, it follows that:

$$i(t) = \frac{9}{2} + \frac{5}{6} e^{-t/0.15}\ \text{mA}$$

Example 3: complex circuit

- Plot of $i(t)$



$$i(t) = \frac{9}{2} + \frac{5}{6} e^{-t/0.15} \text{ mA}$$

Step response

Step response

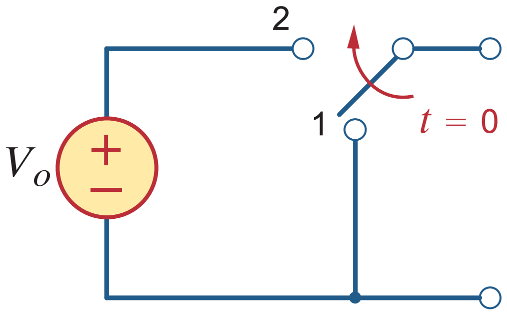
- Examples thus far: abruptly turning on or off a voltage or a current source

- Mathematical model: a unit step function

$$u(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ 1 & \text{for} \quad t > 0 \end{cases}$$

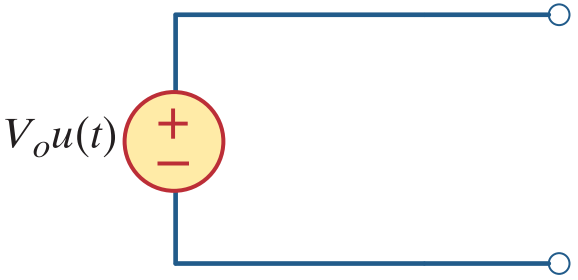
Step response

- In a circuit: a voltage source + a switch



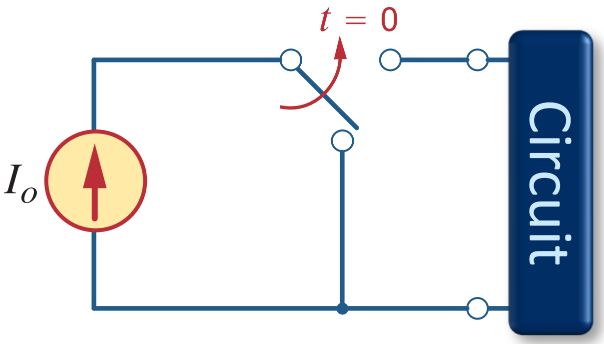
- = a voltage source $V_0 u(t)$:

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$



Step response

- In a circuit: a current source + a switch

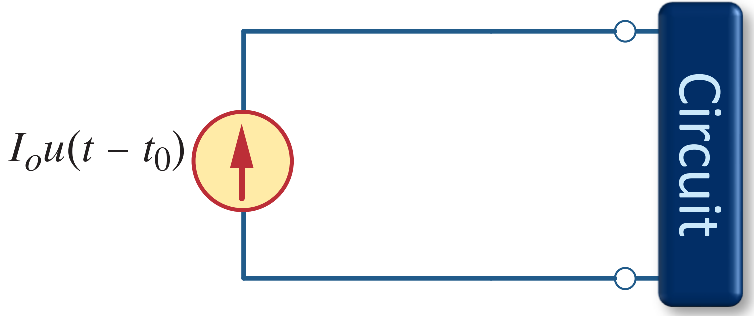


- = a current source $I_0 u(t)$:

$$u(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t > 0 \end{cases}$$

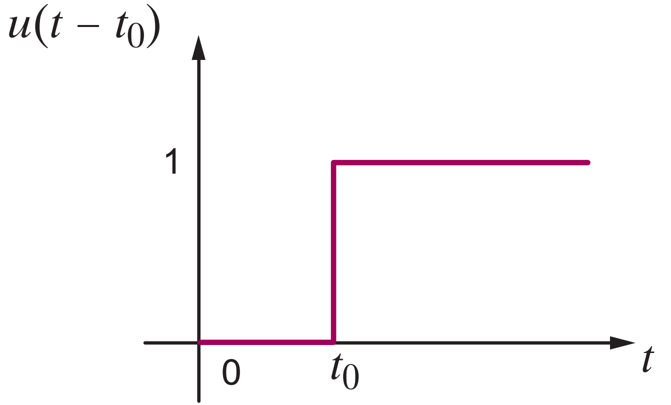
Step response

- A time shift can be added:



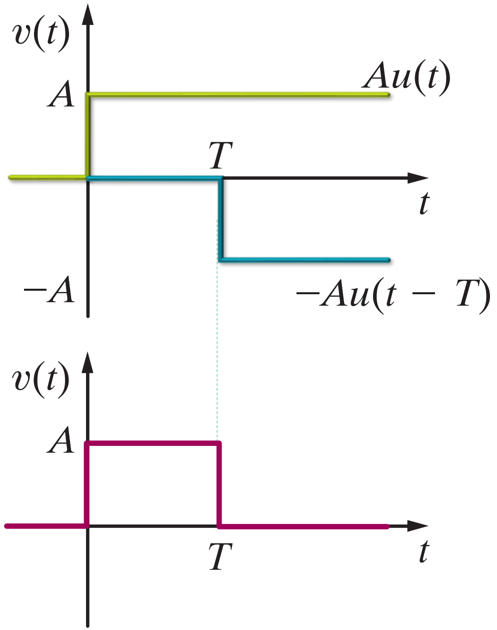
- The unit step function becomes:

$$u(t-t_0)=\begin{cases}0 & \text{for } t<t_0\\ 1 & \text{for } t>t_0\end{cases}$$



Step response

- With two unit step functions one can synthesise a pulse



$$v(t) = A\left[u(t) - u(t-T)\right]$$

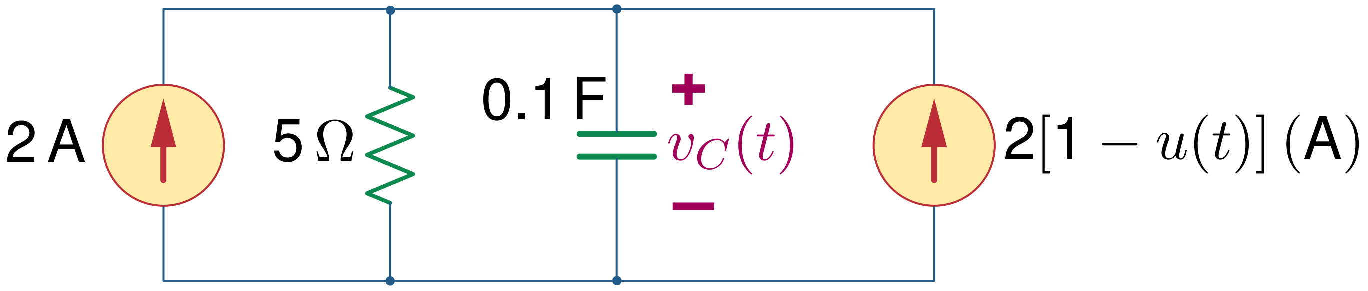
Or time-shifted by t_0:

$$v(t) = A\left\{u(t-t_0) - u\left[t-(t_0+T)\right]\right\}$$

Exam exercise example

Exam(ple)

- Consider the following circuit:



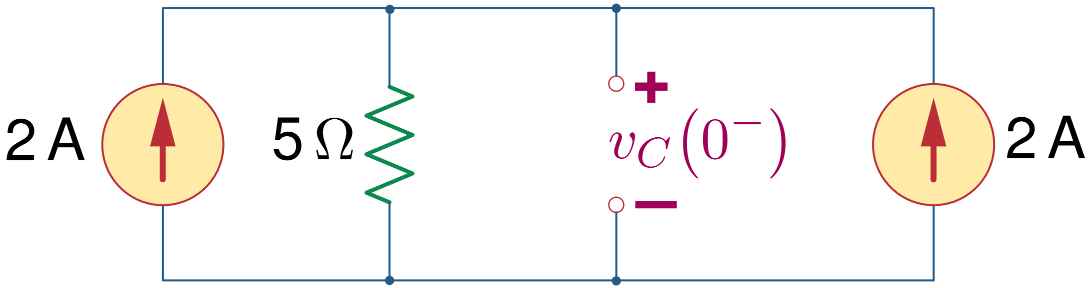
a) Determine $v_C(0+)$.

b) Determine $v_C(t)$ for $t > 0$ s.

Exam(ple)

a) Determine $v_C(0+)$.

- Redraw the circuit for $t < 0$:



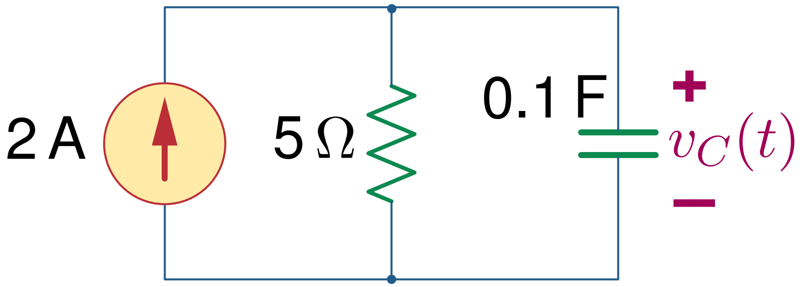
- Calculate the voltage $v_C(0^-)$: $v_C(0^-) = (2+2)5 = 20\,(\text{V})$
- Apply the continuity conditions at $t = 0$:

$$v_C(0^+) = v_C(0^-) = 20\,(\text{V})$$

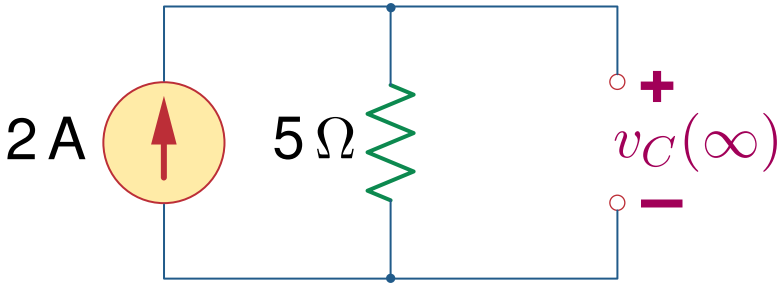
Exam(ple)

b) Determine $v_C(t)$ for $t > 0$ s.

- Redraw the circuit for $t > 0$:

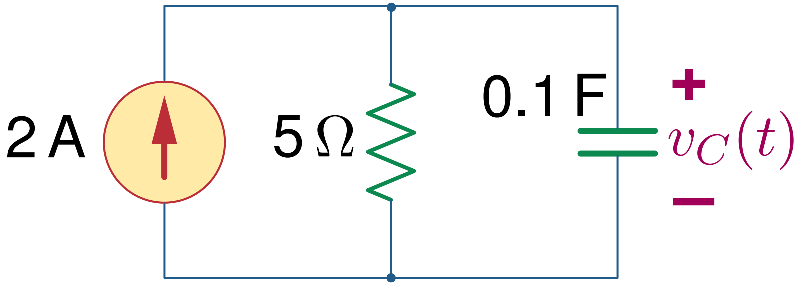


- Redraw the circuit for $t \to \infty$:



- Calculate $v_C(\infty)$: $v_C(\infty) = 2 \cdot 5 = 10\,(\mathrm{V})$

Exam(ple)



b) Determine $v_C(t)$ for $t > 0$ s.

- Calculate the Thévenin resistance: $R_{\mathrm{Th}} = 5\,\Omega$
- Calculate the time constant: $\tau = R_{\mathrm{Th}}C = 0.5\,(\mathrm{s})$
- Assemble vC(t) via the initial-final values formula:

$$v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]\exp(-t/\tau)$$
$$= 10\left(1 + e^{-2t}\right)\ (\mathrm{V})$$

Summary of the day

- First-order transient circuits:
  - always one capacitance or one inductance
  - can contain more resistances and/or sources
- The solution can be derived via:
  - differential equation techniques + identifying coefficients
  - an algorithm
- Step response:
  - step function $u(t)$
  - pulses can be assembled by combining 2 step functions

Next tasks

- Please do the SGH6
- Seminars of Tuesday and Friday
- Register for the end-of-term exam!
- Next week: second-order circuits

Thank you!

Your opinion counts!

- The course management and we are highly interested in your feedback on the course → let us know what you think about the methods of education, assessment, organisation, etc.
- We are also interested in your opinion on the Circuits Course Labs
- Please go to: https://evasys-survey.tudelft.nl/evasys/online.php?p=4EXUN,

or scan the QR-code

Many thanks in advance for your feedback!

The results will be published on Brightspace
Questions: QualityAssurance-EEMCS@tudelft.nl

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