EE1C2 “Linear Circuits B”

Week 2.8

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Mathematics and Computer Science

Today

- Recapitulation of week 2.7
- New topics:
  - Circuit analysis via Laplace transform
    - equivalent circuits in the Laplace-domain (s-domain)
    - circuit analysis in the Laplace-domain
  - Transfer functions in the Laplace-domain
- Summary of the day
- Next time

Recap of week 2.7

- Direct Laplace transform & properties
- Inverse Laplace transform
- Algebraic (+ table lookup) inversion method → requisite skills:
  - familiarity with the Laplace transform properties
  - familiarity with the table of Laplace transforms
  - basic algebra: breaking down fractions

Recap of week 2.7

$V_{\text{in}}(t) = A\cos(\omega t + \phi)$ $\xrightarrow{\mathcal{L}[V_{\text{in}}(t)]}$ $\boldsymbol{V}_{\text{in}} = A\exp(\mathrm{j}\phi)$

$$\frac{\mathrm{d}^3V}{\mathrm{d}t^3} + a\frac{\mathrm{d}^2V}{\mathrm{d}t^2} + b\frac{\mathrm{d}V}{\mathrm{d}t} + cV = f(t)$$

Trigonometry

$$(\mathrm{j}\omega)^3\boldsymbol{V} + a(\mathrm{j}\omega)^2\boldsymbol{V} + b(\mathrm{j}\omega)\boldsymbol{V} + c\boldsymbol{V} = \boldsymbol{F}$$

Complex ? algebra

$V_{\text{out}}(t) = B\cos(\omega t + \theta)$ $\xleftarrow{\mathcal{L}^{-1}[\boldsymbol{V}_{\text{out}}(s)]}$ $\boldsymbol{V}_{\text{out}} = B\exp(\mathrm{j}\theta)$

Recap of week 2.7

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$

Definition:

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt \quad \text{with} \quad s = \sigma + j\omega$$

Recap of week 2.7

Quantity		Measure unit
Time-domain		
time	t	s
voltage	$v(t)$	V
current	$i(t)$	A
Laplace-domain	$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$ with $s = \sigma + j\omega$	
Laplace variable	s	s^{-1}
voltage	$\mathbf{V}(s)$	V·s
current	$\mathbf{I}(s)$	A·s

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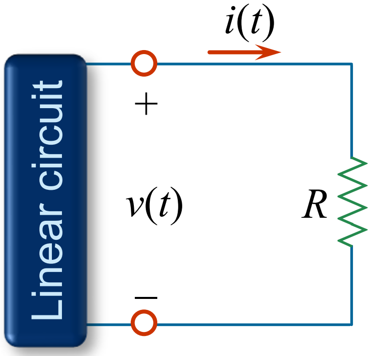
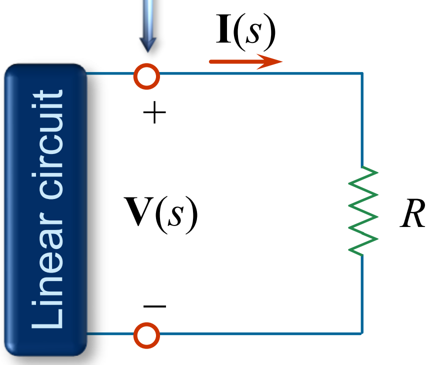
Circuit analysis via Laplace transform

Laplace transform & circuit elements:
the resistance

$$v(t) = Ri(t)$$

$$\boxed{\mathbf{V}(s) = R\mathbf{I}(s)}$$





Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$

Laplace transform & circuit elements:
the capacitance

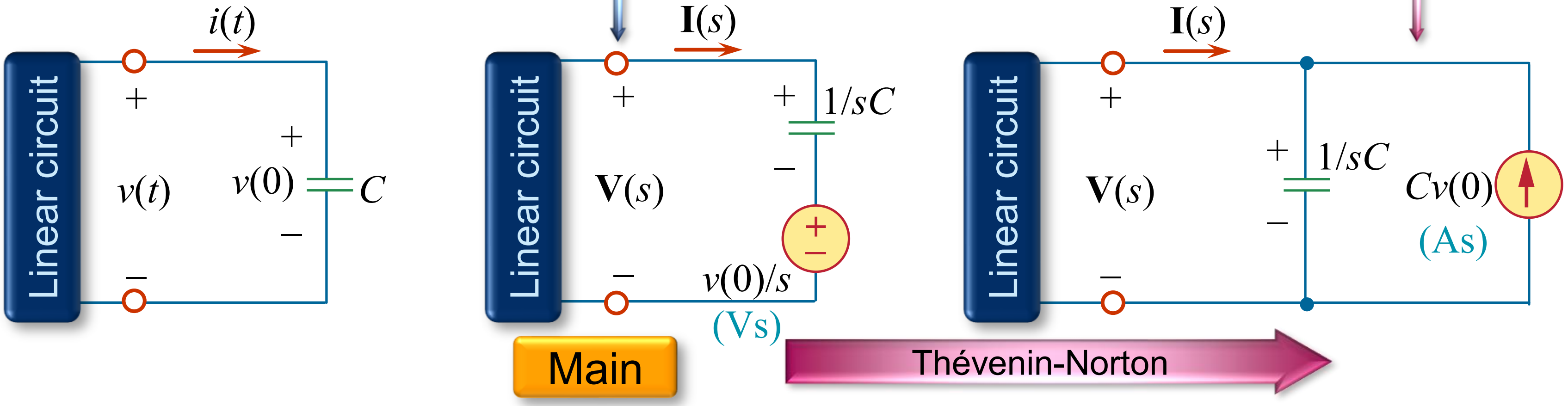
$$\int_0^t f(x)\,dx \Longleftrightarrow \frac{1}{s}F(s) + u(t) \Longleftrightarrow \frac{1}{s} \qquad \frac{df}{dt} \Longleftrightarrow sF(s) - f(0^-)$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$$

$$\boxed{\mathbf{V}(s) = \frac{\mathbf{I}(s)}{sC} + \frac{v(0)}{s}}$$

$$\boxed{\mathbf{I}(s) = sC\mathbf{V}(s) - Cv(0)}$$



Laplace transform & circuit elements:
the inductance

$$\frac{df}{dt} \Longleftrightarrow sF(s) - f(0^-)$$

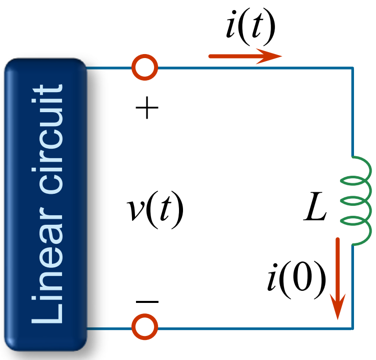
$$\int_0^t f(x)\,dx \Longleftrightarrow \frac{1}{s}F(s) \quad + \quad u(t) \Longleftrightarrow \frac{1}{s}$$

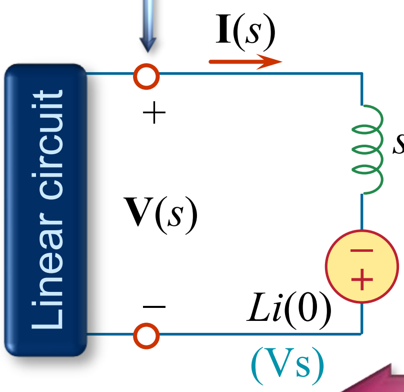
$$v(t) = L\frac{di(t)}{dt}$$

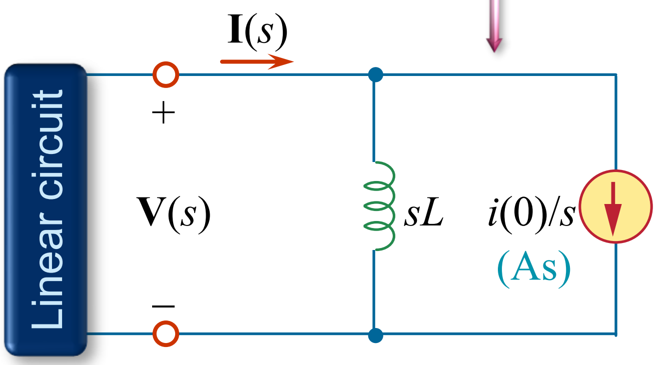
$$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0)$$

$$\boxed{\mathbf{V}(s) = sL\mathbf{I}(s) - Li(0)}$$

$$\boxed{\mathbf{I}(s) = \frac{\mathbf{V}(s)}{sL} + \frac{i(0)}{s}}$$







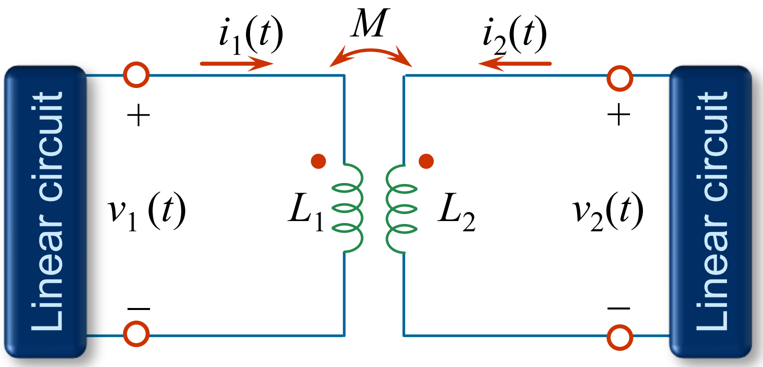
Norton-Thévenin

Main

Laplace transform & circuit elements: magnetically coupled inductances

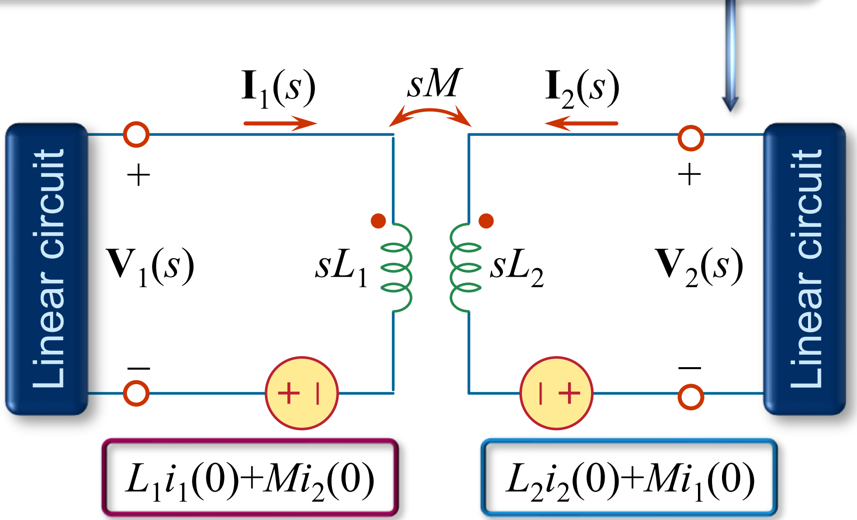
$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = L_2\frac{di_2(t)}{dt} + M\frac{di_1(t)}{dt}$$



$$\mathbf{V}_1(s) = sL_1\mathbf{I}_1(s) - L_1 i_1(0) + sM\mathbf{I}_2(s) - Mi_2(0)$$

$$\mathbf{V}_2(s) = sL_2\mathbf{I}_2(s) - L_2 i_2(0) + sM\mathbf{I}_1(s) - Mi_1(0)$$



Laplace transform & circuit elements: problem-solving strategy

- Step 1: Assume that the circuit has reached steady state before a switch is moved:
  i. draw the circuit valid for $t = 0^-$ replacing capacitors with open circuits and inductors with short circuits;
  ii. solve for the initial conditions: voltages across capacitor and currents flowing through inductor

  Remember that $v_C(0^-) = v_C(0^+) = v_C(0)$ and $i_L(0^-) = i_L(0^+) = i_L(0)$
- Step 2: Draw the circuit for $t > 0$ $\Longrightarrow$ use the equivalent circuits for circuit elements $\Longleftrightarrow$ writing Laplace-domain transformed, integro-differential equations

Laplace transform & circuit elements:

problem-solving strategy

- Step 3: Use circuit analysis techniques for analysing the resulting circuit → Laplace-domain algebraic equations

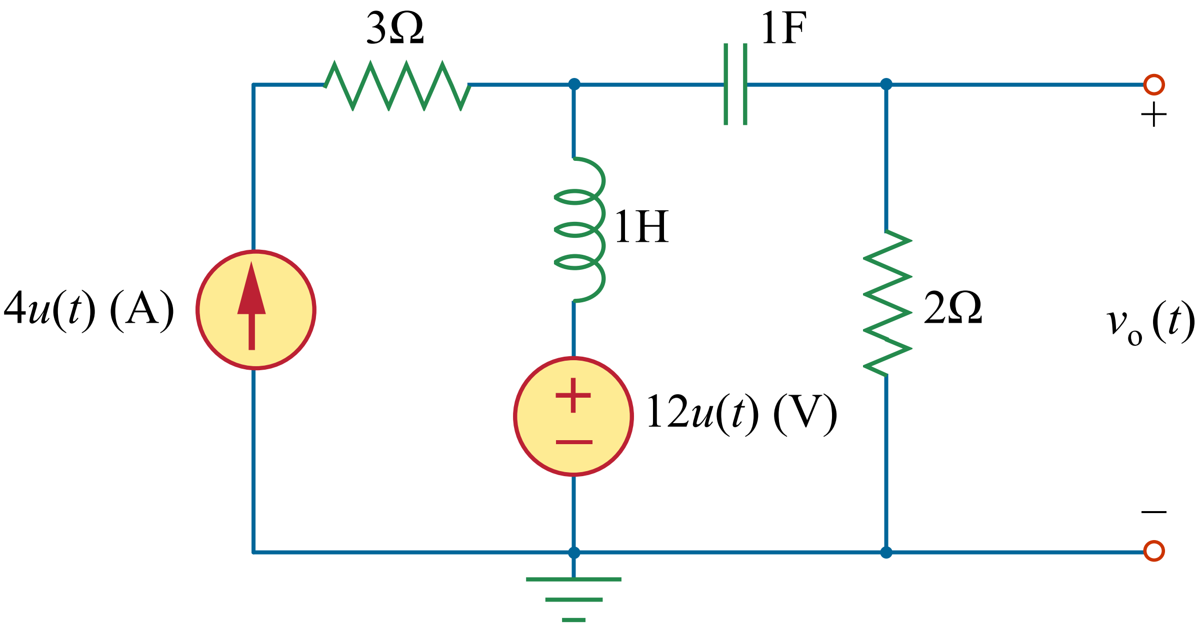
- Step 4: Solve the algebraic equations for the variable of interest → the result will be a ratio of polynomials in the complex variable s

- Step 5: Perform an inverse Laplace transform to solve for the circuit response in the time-domain → revisit the algebraic + table lookup inversion steps!!!

La Persistance de la mémoire

Analysis: example

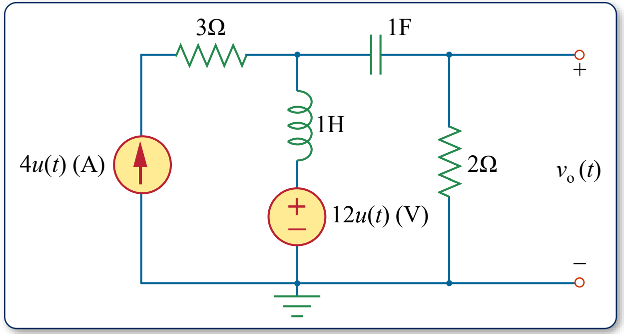
- Determine $v_o(t)$ in the circuit:

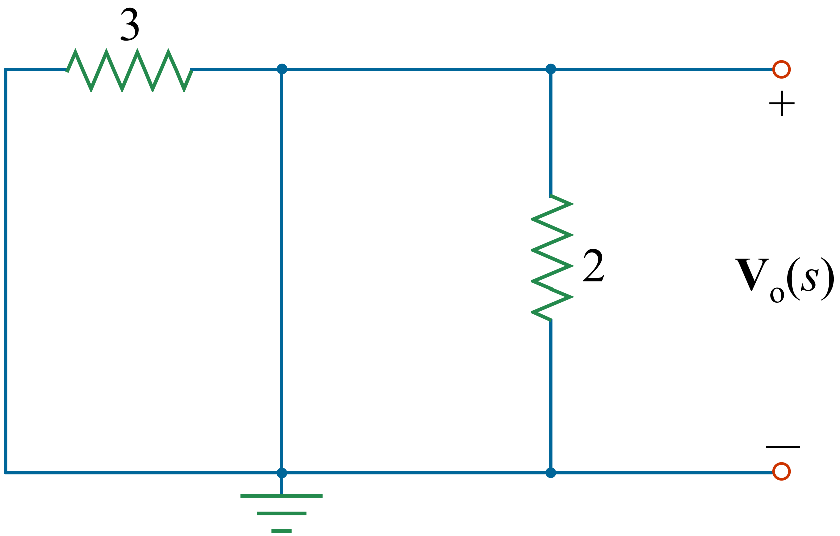


Analysis: example

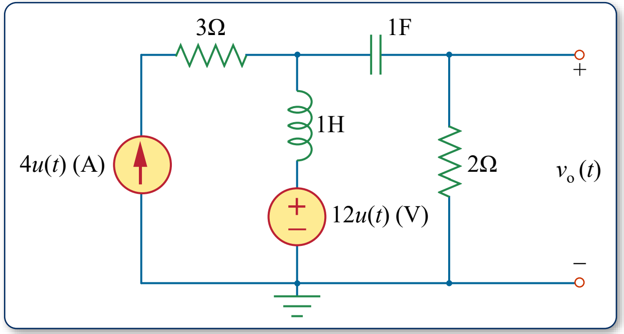
Laplace-domain circuit

Resistances





Analysis: example

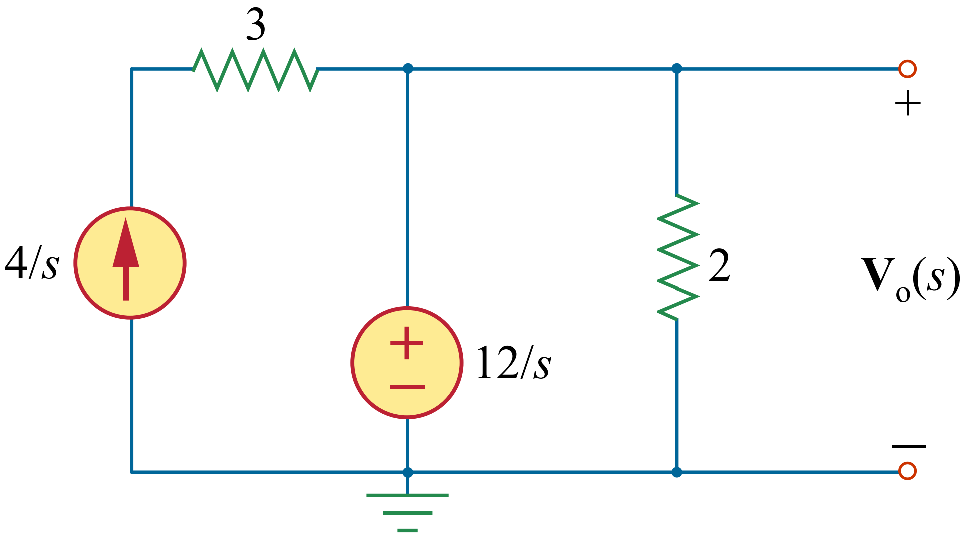


Laplace-domain circuit

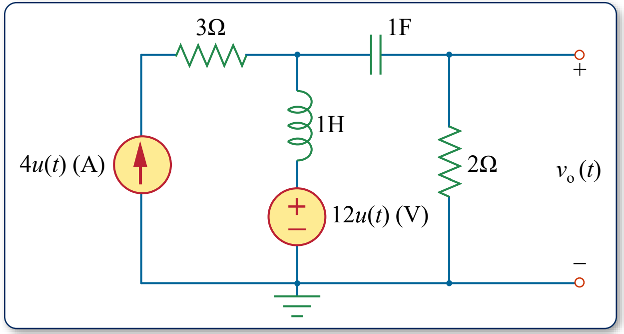
Sources

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$

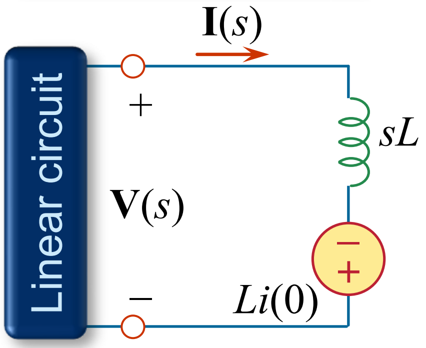


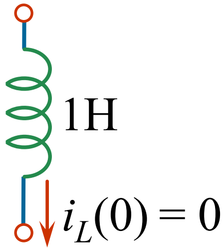
Analysis: example

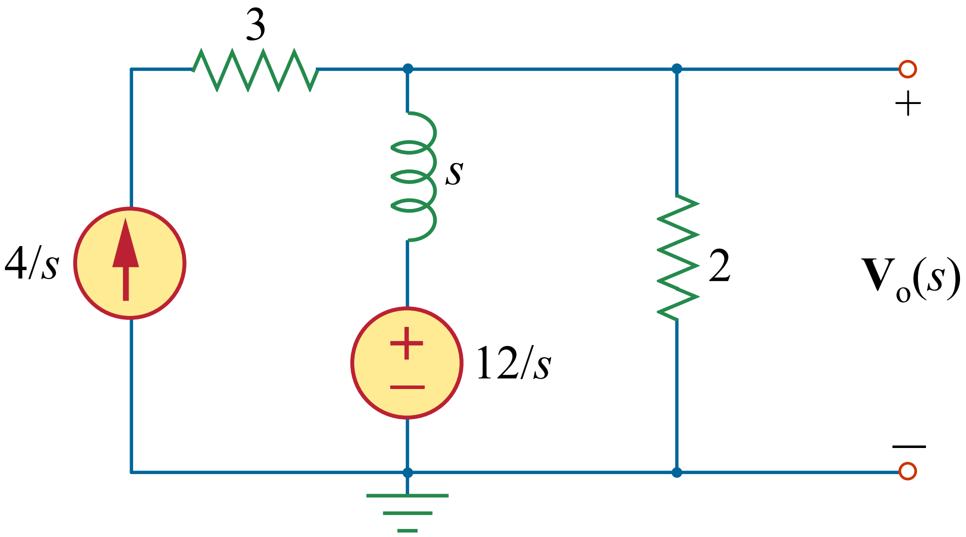


Laplace-domain circuit

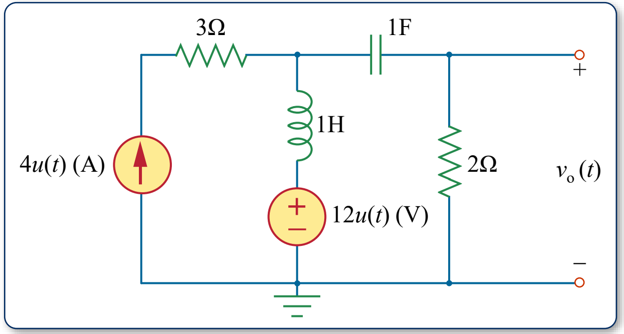
Inductances





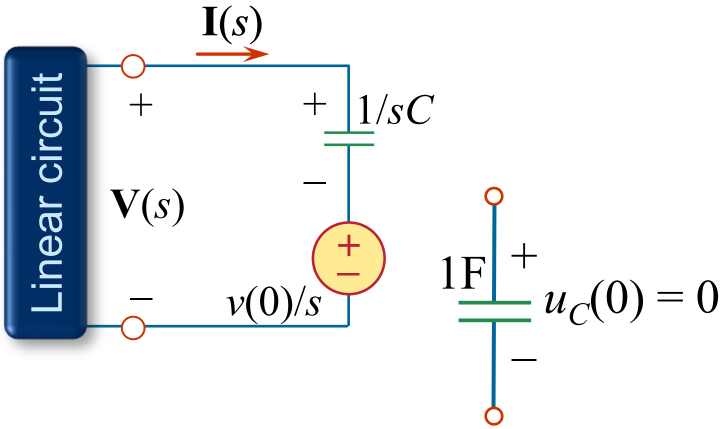


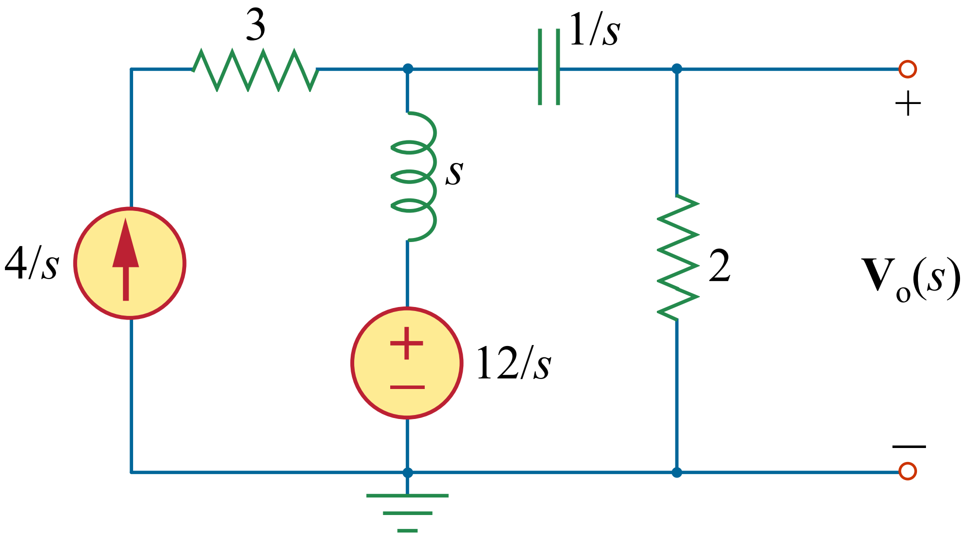
Analysis: example



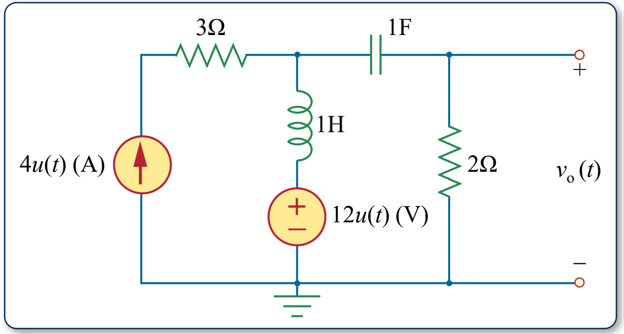
Laplace-domain circuit

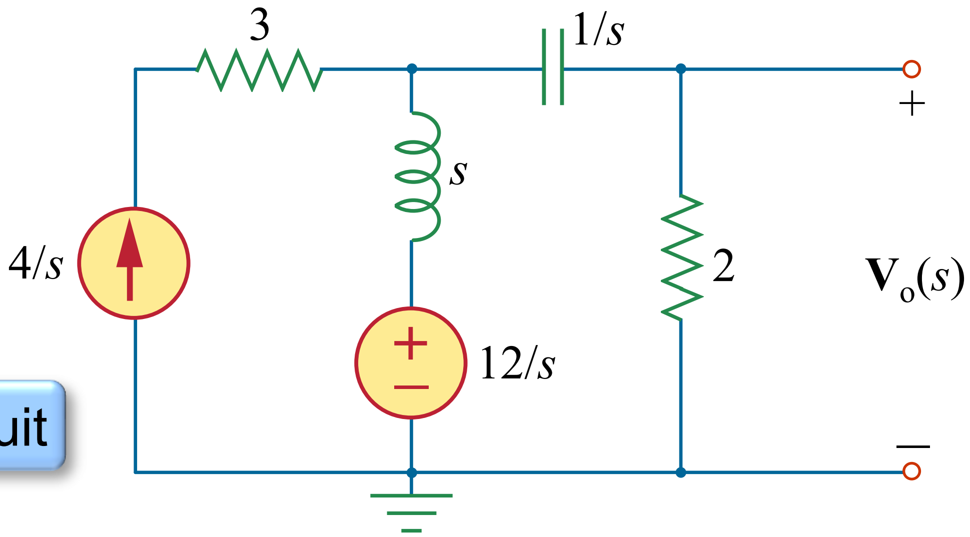
Capacitances





Analysis: example

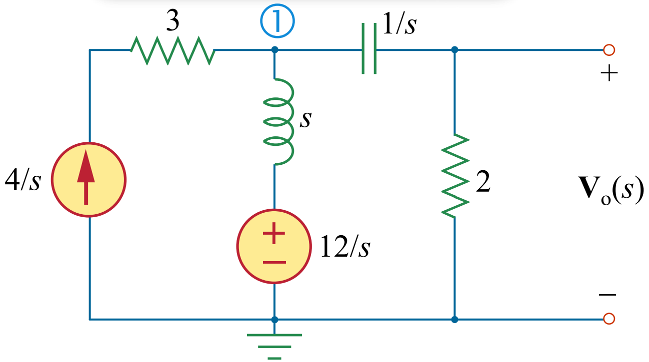




Assembled Laplace-domain circuit

Analysis: example

Nodal analysis

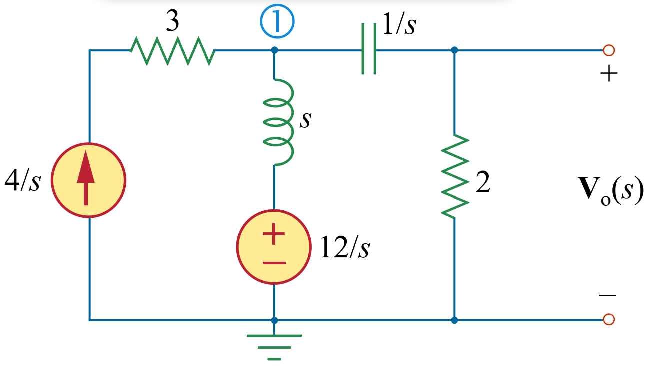


KCL at node ①

$$-\frac{4}{s}+\frac{\mathbf{V}_1(s)-\frac{12}{s}}{s}+\frac{\mathbf{V}_1(s)}{\frac{1}{s}+2}=0$$

Analysis: example

Nodal analysis



KCL at node ①

$$-\frac{4}{s}+\frac{\mathbf{V}_1(s)-\frac{12}{s}}{s}+\frac{\mathbf{V}_1(s)}{\frac{1}{s}+2}=0$$

$$\Downarrow$$

$$\mathbf{V}_1(s)=\frac{4(s+3)(2s+1)}{s(s^2+2s+1)}$$

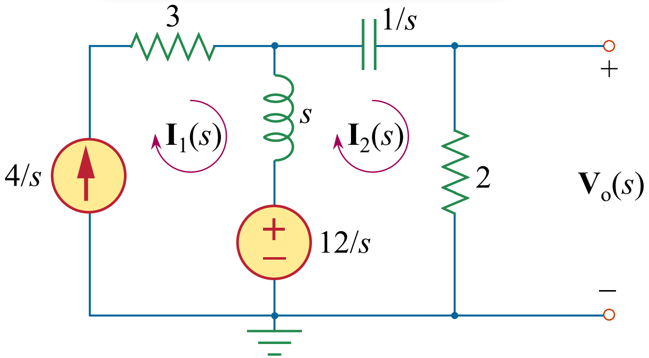
Apply voltage division

$$\mathbf{V}_o(s)=\mathbf{V}_1(s)\left(\frac{2}{\frac{1}{s}+2}\right)=\mathbf{V}_1(s)\left(\frac{2s}{2s+1}\right)$$

$$\boxed{=\frac{8(s+3)}{(s+1)^2}}$$

Analysis: example

Mesh analysis



KVL on mesh $\mathbf{I}_1(s)$

$$\mathbf{I}_1(s) = \frac{4}{s}$$

KVL on mesh $\mathbf{I}_2(s)$

$$\frac{12}{s} - [\mathbf{I}_2(s) - \mathbf{I}_1(s)]s - \frac{\mathbf{I}_2(s)}{s} - 2\mathbf{I}_2(s) = 0$$

substitute $\mathrm{I}_1(s)$

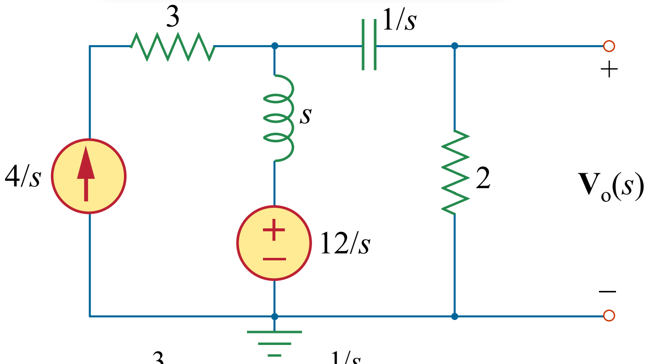
$$\mathbf{I}_2(s) = \frac{4(s+3)}{(s+1)^2}$$

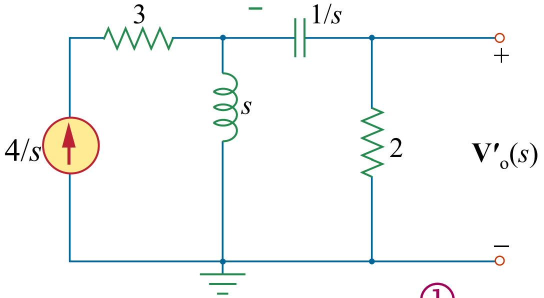
account for the 2Ω resistance

$$\boxed{\mathbf{V}_o(s) = \frac{8(s+3)}{(s+1)^2}}$$

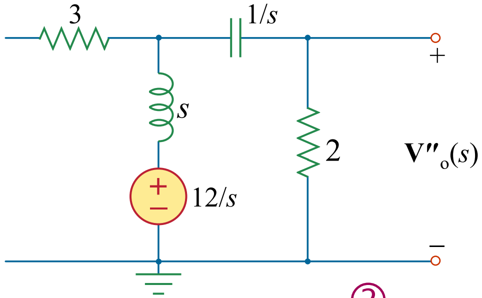
Analysis: example

Superposition





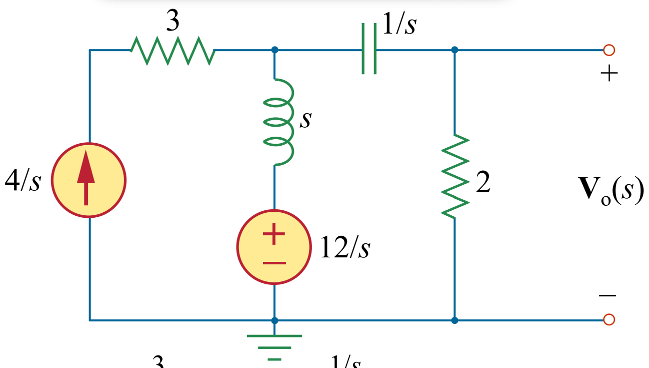
①

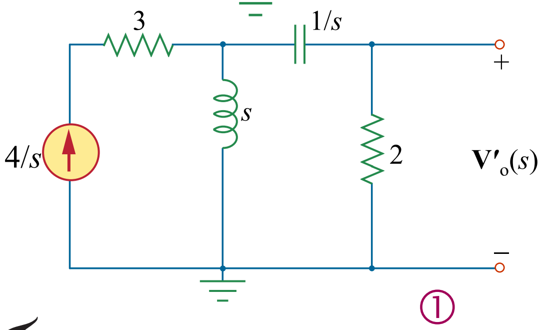


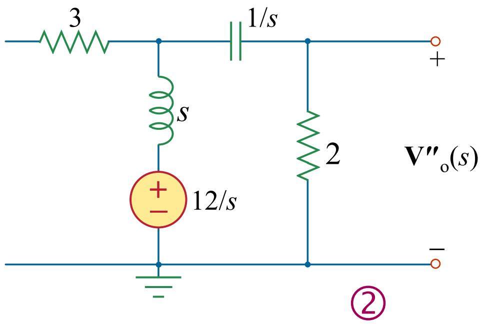
②

Analysis: example

Superposition







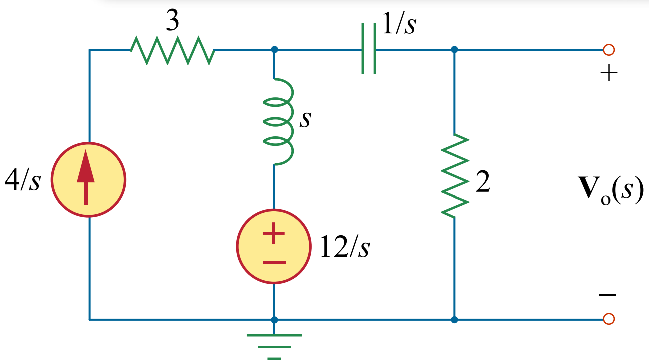
$$①:\ \mathbf{V}'_o(s) = \left[\frac{2\frac{4}{s}s}{s+\frac{1}{s}+2}\right] = \frac{8s}{s^2+2s+1}$$

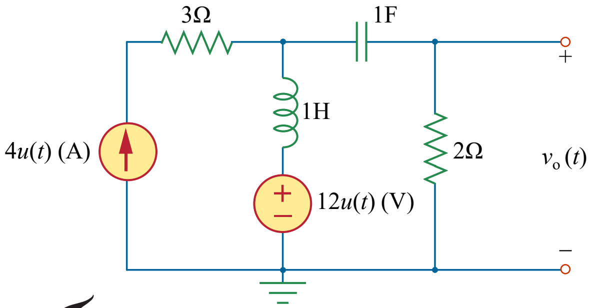
$$②:\ \mathbf{V}''_o(s) = \left[\frac{\frac{12}{s}2}{s+\frac{1}{s}+2}\right] = \frac{24}{s^2+2s+1}$$

$$\mathbf{V}_o(s) = \mathbf{V}'_o(s) + \mathbf{V}''_o(s) \boxed{= \frac{8(s+3)}{(s+1)^2}}$$

Analysis: example

Inversion to time-domain





$$\mathbf{V}_o(s) = \frac{8(s+3)}{(s+1)^2}$$

$$= \frac{K_{11}}{(s+1)^2} + \frac{K_{12}}{s+1}$$

$$= \frac{16}{(s+1)^2} + \frac{8}{s+1}$$

$$\boxed{v_o(t) = \left(16te^{-t} + 8e^{-t}\right)u(t) \text{ V}}$$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{s}$
e^{-at}	$\dfrac{1}{s+a}$
t	$\dfrac{1}{s^2}$

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$

Transfer functions in the Laplace-domain

Transfer functions

- Laplace domain definition:

$$\mathbf{H}(s)=\frac{\mathbf{Y}_o(s)}{\mathbf{X}_i(s)}$$

$$\Downarrow$$

$$\mathbf{Y}_o(s)=\mathbf{H}(s)\cdot\mathbf{X}_i(s)$$

- Particular cases:
  - unit impulse response:

    $\boldsymbol{X}_{\mathrm{i}}(s)=1 \longrightarrow x_{\mathrm{i}}(t)=\delta(t)$
  - time-domain response:

    $y_{\mathrm{o}}(t)=h(t)$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
⋮		
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s}F(s)$
⋮		
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Transfer functions

- Laplace domain definition:

$$\mathbf{H}(s) = \frac{\mathbf{Y}_o(s)}{\mathbf{X}_i(s)}$$

$$\Downarrow$$

$$\mathbf{Y}_o(s) = \mathbf{H}(s) \cdot \mathbf{X}_i(s)$$

- Particular cases:
  - step response:

$$\boldsymbol{X}_{\mathrm{i}}(s) = 1/s \longrightarrow \boldsymbol{Y}_{\mathrm{o}}(s) = \boldsymbol{H}(s)/s$$

  - time-domain response:

$$y_{\mathrm{o}}(t) = h(t) * u(t) = \int_0^t h(\tau)\mathrm{d}\tau$$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
$\vdots$		
Time integration	$\int_0^t f(x)\,dx$	$\frac{1}{s}F(s)$
$\vdots$		
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Transfer functions

- Laplace domain definition:

$$\mathbf{H}(s) = \frac{\mathbf{Y}_o(s)}{\mathbf{X}_i(s)}$$

$$\Downarrow$$

$$\mathbf{Y}_o(s) = \mathbf{H}(s) \cdot \mathbf{X}_i(s)$$

- Particular cases:
  – step response:

  $$\boldsymbol{X}_\mathrm{i}(s) = 1/s \longrightarrow \boldsymbol{Y}_\mathrm{o}(s) = \boldsymbol{H}(s)/s$$

  – time-domain response:

  $$y_\mathrm{o}(t) = h(t)*u(t) = \int_0^t h(\tau)\mathrm{d}\tau$$

The transfer function provides critical information on the operation and the stability of a system

Stable systems:
All poles and zeros lie in the left half of the s-plane

Recap: characteristic equation

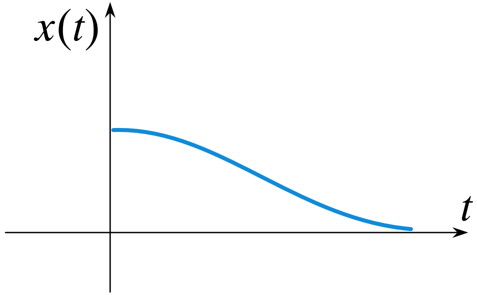
Case 1: overdamped circuit

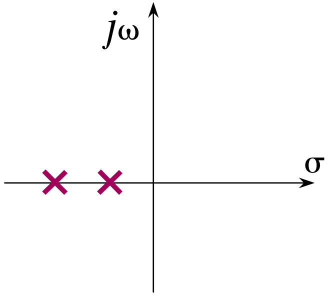
- $s_1 \neq s_2$, and s_1 and s_2 are real $\longrightarrow F(s) = \dfrac{k_1}{s + s_1} + \dfrac{k_2}{s + s_2}$
- discriminant: $b^2\text{-}4c > 0$

simple poles

$$s_1, s_2 = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

$$\boxed{x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + K_3}$$





Recap: characteristic equation

Case 2: underdamped circuit

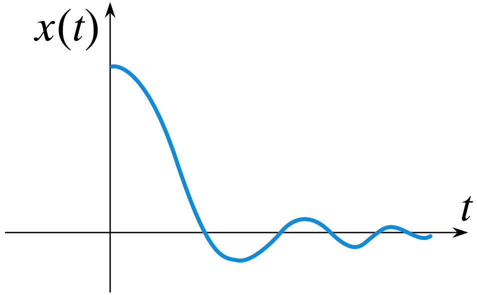
- $s_1 \neq s_2$, and s_1 and s_2 are complex $\longrightarrow$ $F(s) = \dfrac{k_1}{s + s_1} + \dfrac{k_2}{s + s_2}$
- discriminant: b^2-4$c < 0$

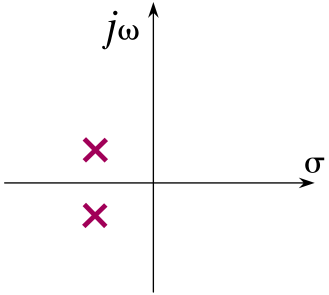
complex conjugate poles

$$s_1, s_2 = -\frac{b}{2} \pm j\frac{\sqrt{4c - b^2}}{2} = -\sigma \pm j\omega_d$$

$$\boxed{x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} + K_3}$$

$$x(t) = e^{-\sigma t}\left[A\cos(\omega_d t) + B\sin(\omega_d t)\right] + k_3$$





Recap: characteristic equation

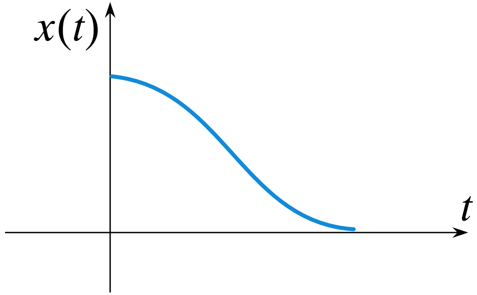
Case 3: critically damped circuit

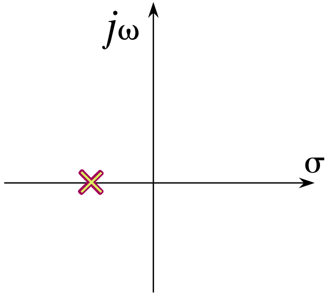
- $s_1 = s_2$, and s_1 and s_2 are real $\longrightarrow F(s) = \dfrac{k_1}{s + s_1} + \dfrac{k_2}{(s + s_1)^2}$
- discriminant: $b^2\text{-}4c = 0$

overlapped poles

$$s_1, s_2 = -\frac{b}{2}$$

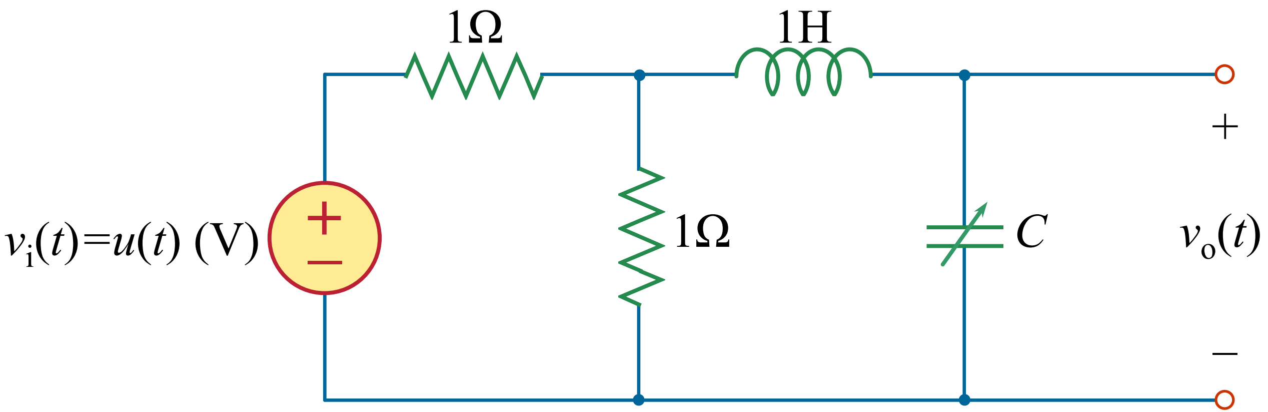
$$x(t) = (K_1 + K_2 t)e^{s_1 t} + K_3$$





Transfer function: example

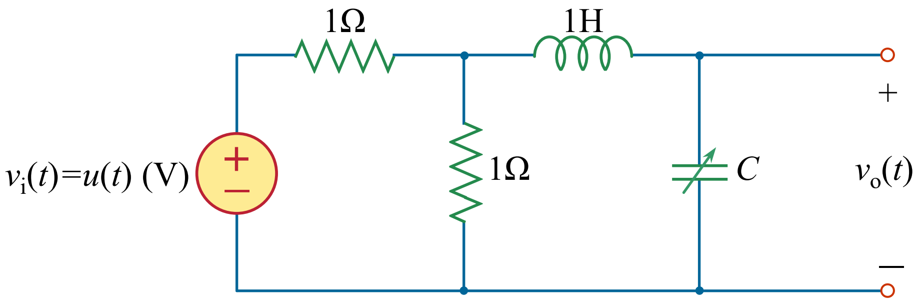
- Determine $\mathbf{V}_o(s)/\mathbf{V}_i(s)$ for the circuit in the figure below and the step response



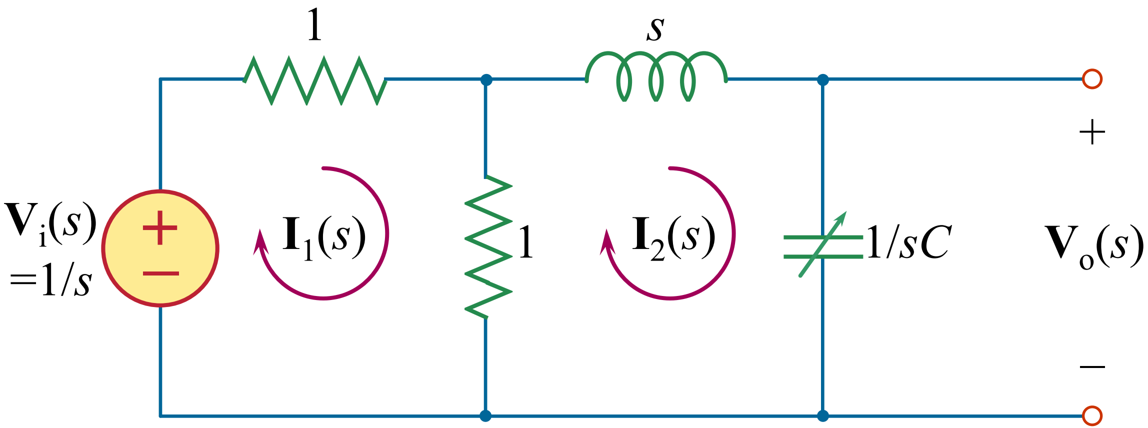
3 cases:

1. $C = 8\text{F}$
2. $C = 16\text{F}$
3. $C = 32\text{F}$

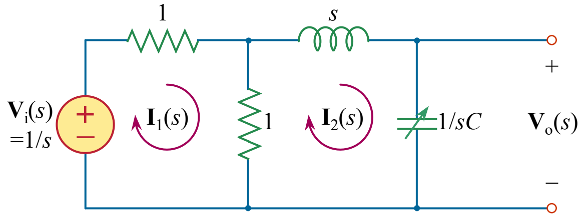
Transfer function: example



Laplace-domain circuit



Transfer function: example



- Transfer function:

$$\mathbf{H}(s)=\frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)}=\frac{\frac{1}{2C}}{s^2+\frac{1}{2}s+\frac{1}{C}}$$

- Step response:

$$\mathbf{V}_o(s)=\frac{\frac{1}{2C}}{s\left(s^2+\frac{1}{2}s+\frac{1}{C}\right)}$$

KVL on mesh $\mathbf{I}_2(s)$

$$-\mathbf{I}_1(s)+\left(s+\frac{1}{sC}+1\right)\mathbf{I}_2(s)=0$$

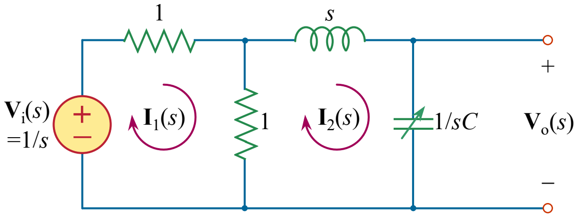
KVL on mesh $\mathbf{I}_1(s)$

$$2\mathbf{I}_1(s)-\mathbf{I}_2(s)=\mathbf{V}_i(s)$$

$$\Downarrow$$

$$\mathbf{V}_o(s)=\frac{1}{sC}\mathbf{I}_2(s)$$

Transfer function: example



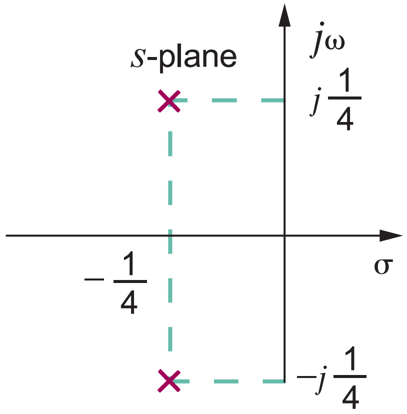
Case 1: $C = 8\mathrm{F}$

- Transfer function:

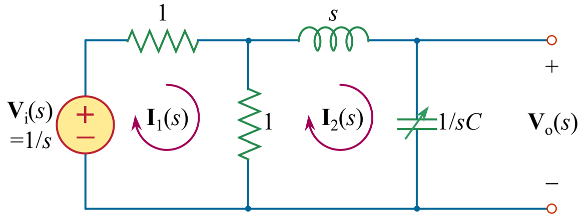
$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}} = \frac{\frac{1}{16}}{s^2 + \frac{1}{2}s + \frac{1}{8}} = \frac{\frac{1}{16}}{\left(s + \frac{1}{4} - \frac{j}{4}\right)\left(s + \frac{1}{4} + \frac{j}{4}\right)}$$

- Step response:

$$\mathbf{V}_o(s) = \frac{\frac{1}{16}}{s\left(s + \frac{1}{4} - \frac{j}{4}\right)\left(s + \frac{1}{4} + \frac{j}{4}\right)}$$



Transfer function: example



Case 1: $C = 8\text{F}$

- Step response: $s \longrightarrow t$

$$\mathbf{V}_o(s) = \frac{\frac{1}{16}}{s\left(s+\frac{1}{4}-\frac{j}{4}\right)\left(s+\frac{1}{4}+\frac{j}{4}\right)}$$

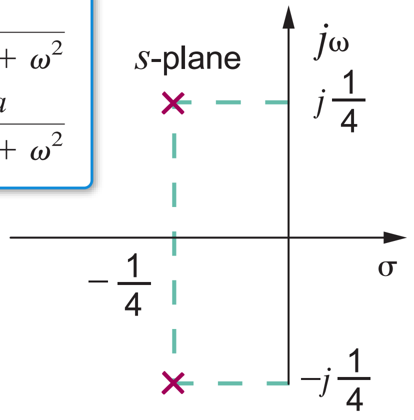
$$= \boxed{\frac{\frac{1}{2}}{s}} + \frac{-\frac{1}{2}\left(s+\frac{1}{4}\right)}{\left(s+\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^2} + \frac{-\frac{1}{2}\left(\frac{1}{4}\right)}{\left(s+\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^2}$$

ω

Also follows from the final value theorem

shift in s-domain $\longrightarrow e^{-at}$

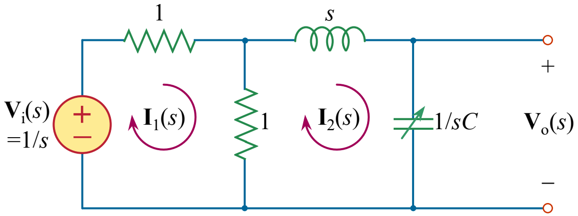
$e^{-at}\sin\omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$



$$v_o(t) = \left[\frac{1}{2} - \frac{1}{2}e^{-t/4}\left(\cos\frac{t}{4}+\sin\frac{t}{4}\right)\right]u(t)\ \text{V}$$

$$= \left[\boxed{\frac{1}{2}} + \frac{1}{\sqrt{2}}e^{-t/4}\cos\left(\frac{t}{4}+135^\circ\right)\right]u(t)\ \text{V}$$

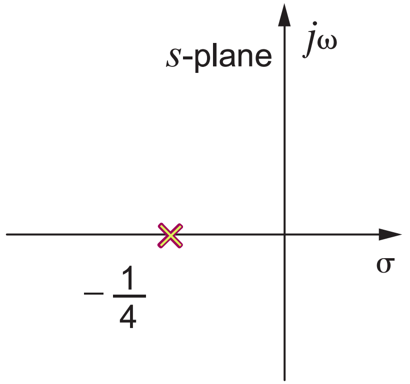
Transfer function: example



Case 2: $C = 16\text{F}$

- Transfer function:

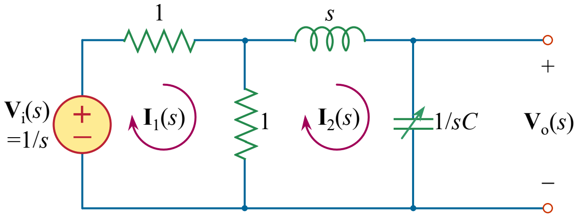
$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}} = \frac{\frac{1}{32}}{s^2 + \frac{1}{2}s + \frac{1}{16}} \boxed{= \frac{\frac{1}{32}}{\left(s + \frac{1}{4}\right)^2}}$$



- Step response:

$$\boxed{\mathbf{V}_o(s) = \frac{\frac{1}{32}}{s\left(s + \frac{1}{4}\right)^2}}$$

Transfer function: example



Case 2: $C = 16\text{F}$

- Step response: $s \longrightarrow t$

$$\mathbf{V}_o(s) = \frac{\frac{1}{32}}{s\left(s+\frac{1}{4}\right)^2}$$

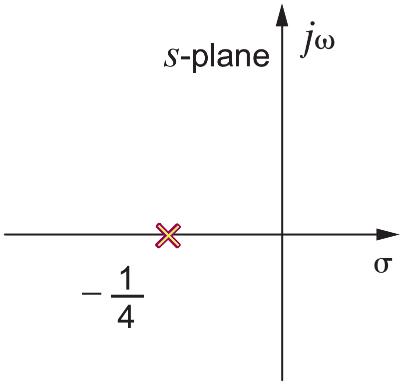
Also follows from the final value theorem

$$= \boxed{\frac{\frac{1}{2}}{s}} + \frac{-\frac{1}{2}}{s+\frac{1}{4}} + \frac{-\frac{1}{8}}{\left(s+\frac{1}{4}\right)^2}$$

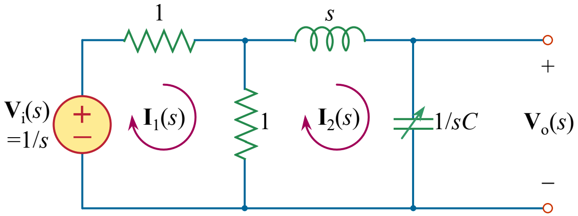
shift in s-domain $\longrightarrow e^{-at}$

$$v_o(t) = \left[\boxed{\frac{1}{2}} - \left(\frac{t}{8}+\frac{1}{2}\right)e^{-t/4}\right]u(t)\ \text{V}$$

e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$



Transfer function: example



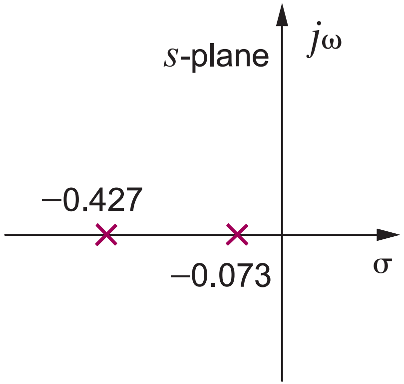
Case 3: $C = 32\text{F}$

- Transfer function:

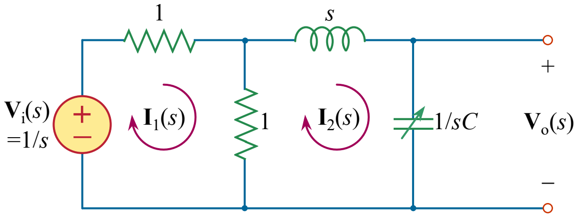
$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{\frac{1}{2C}}{s^2 + \frac{1}{2}s + \frac{1}{C}} = \frac{\frac{1}{64}}{s^2 + \frac{1}{2}s + \frac{1}{32}} = \frac{\frac{1}{64}}{(s+0,427)(s+0,073)}$$

- Step response:

$$\mathbf{V}_o(s) = \frac{\frac{1}{64}}{s(s+0,427)(s+0,073)}$$



Transfer function: example



Case 3: $C = 32\text{F}$

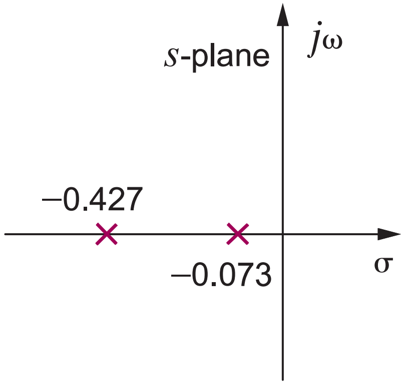
- Step response: $s \longrightarrow t$

$$\mathbf{V}_o(s) = \frac{\frac{1}{64}}{s(s+0,427)(s+0,073)}$$

$$= \boxed{\frac{\frac{1}{2}}{s}} + \frac{0,103}{s+0,427} + \frac{-0,603}{s+0,073}$$

Also follows from the final value theorem

e^{-at}	$\dfrac{1}{s+a}$

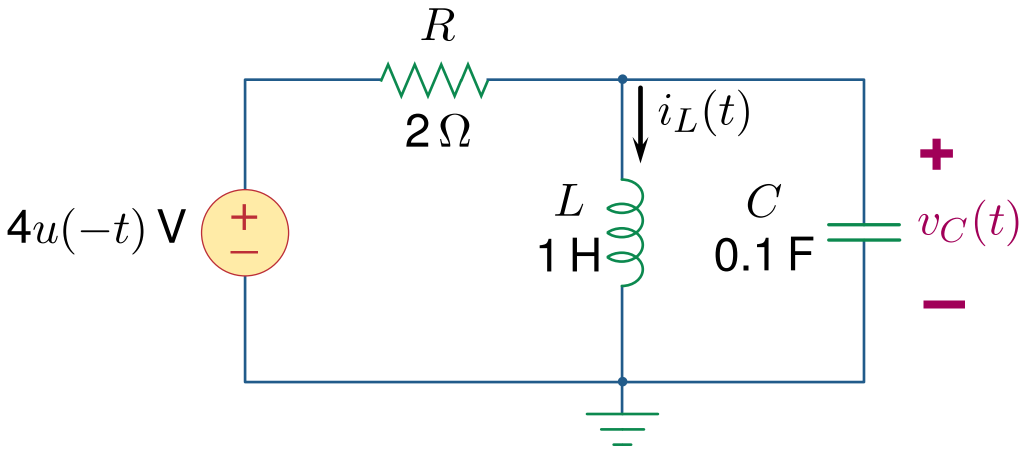


$$v_o(t) = \left[\boxed{\frac{1}{2}} + 0,103e^{-0,427t} - 0,603e^{-0,073t}\right] u(t)\ \text{V}$$

Exam exercise example

Exam(ple)

Consider the following circuit:



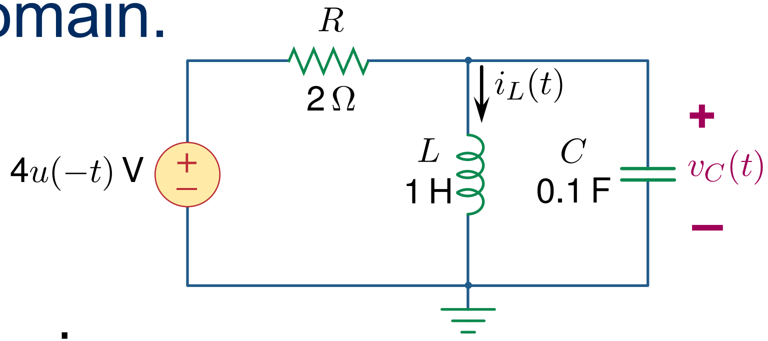
a) Redraw the circuit in the s-domain.

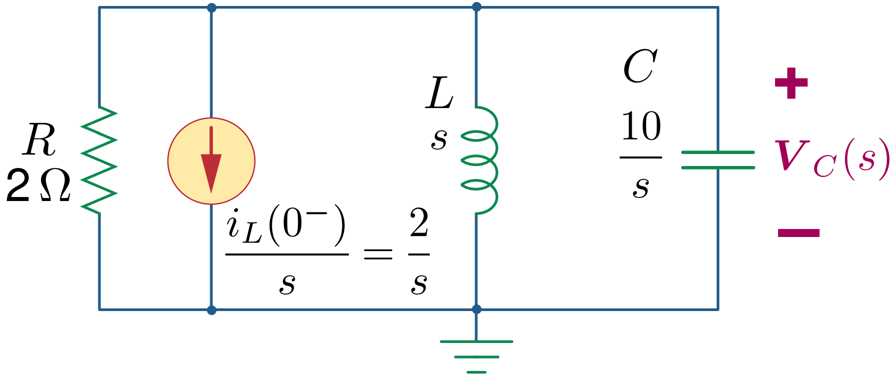
b) Determine $v_C(t)$ by means of the Laplace transform.

Exam(ple)

a) Redraw the circuit in the s-domain.

- Analyse the circuit at $t < 0$:
  - $v_C(0^-) = 0\,(\mathrm{V})$
  - $i_L(0^-) = \dfrac{4}{2} = 2\,(\mathrm{A})$
- Redraw the circuit in the s-domain.

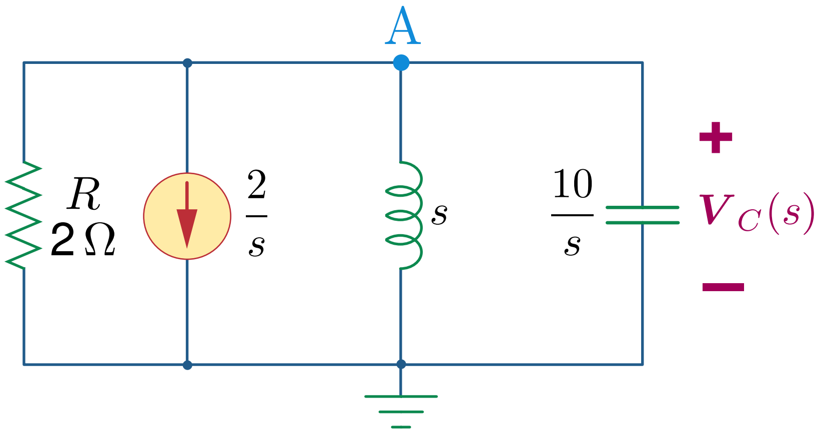




Exam(ple)

b) Determine $v_C(t)$ by means of the Laplace transform.

- Apply KCL @ A:
  - $$\frac{\boldsymbol{V}_C}{2} + \frac{\boldsymbol{V}_C}{s} + \frac{\boldsymbol{V}_C}{10/s} + \frac{2}{s} = 0$$
  - $$\boldsymbol{V}_C \left(\frac{1}{2} + \frac{1}{s} + \frac{s}{10} \right) = -\frac{2}{s}$$
  - $$\boldsymbol{V}_C = -\frac{20}{s^2 + 5s + 10}$$



Exam(ple)

$$\boldsymbol{V}_C = -\frac{20}{s^2 + 5s + 10}$$

b) Determine $v_C(t)$ by means of the Laplace transform.

$e^{-at}\sin\omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$

- Inverse Laplace transform:
  - denominator's discriminant: $\Delta = 25 - 40 < 0$
  - completing the square in the denominator:

$$s^2 + 5s + 10 = (s+2.5)^2 + 3.75 = (s+2.5)^2 + \left(\frac{\sqrt{15}}{2}\right)^2$$

  - $$\boldsymbol{V}_C = -\frac{20}{(s+\underbrace{2.5}_{a})^2 + \left(\underbrace{\frac{\sqrt{15}}{2}}_{\omega}\right)^2} = -\frac{20}{\frac{\sqrt{15}}{2}}\,\frac{\frac{\sqrt{15}}{2}}{(s+2.5)^2 + \left(\frac{\sqrt{15}}{2}\right)^2}$$

Exam(ple)

$$\boldsymbol{V}_C = -\frac{20}{s^2 + 5s + 10}$$

b) Determine $v_C(t)$ by means of the Laplace transform.

- Inverse Laplace transform:
  - denominator's discriminant: $\Delta = 25 - 40 < 0$
  - completing the square in the denominator:

$$s^2 + 5s + 10 = (s + 2.5)^2 + 3.75 = (s + 2.5)^2 + \left(\frac{\sqrt{15}}{2}\right)^2$$

$e^{-at}\sin \omega t$	$\dfrac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at}\cos \omega t$	$\dfrac{s+a}{(s+a)^2 + \omega^2}$

- Inverse Laplace transform:

$$v_C(t) = -\frac{40}{\sqrt{15}} e^{-2.5t} \sin\left(\frac{\sqrt{15}}{2}t\right) u(t) \text{ (V)} = -10.33\, e^{-2.5t} \sin(1.94t) u(t) \text{ (V)}$$

Summary of the day

- Equivalent circuits in the Laplace-domain
- Circuit analysis in the Laplace-domain (actually, not new)
- Transfer functions in the Laplace-domain (technically, not new, but new insights)

Next tasks

- Please do the SGH6
- Seminars on Tuesday and Friday
- Next week: Two-ports

Thank you!